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by

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Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

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Abstract

An experimental investigation was undertaken to determine the effects of an impulsive load of very short duration on a simple structure such as a portal frame. Models built and tests conducted are described. Test results are tabulated and graphically compared with calculations based on a simple theory.

The structural models show greater resistance to deformation than predicted by theory. This increase can most easily be explained by an increase in the yield point due to the dynamic loading. The resistance the models develop, however, is less than that which the results of high strain rate tests would indicate.

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Notation

F, f = Any force

h_1 = Initial height of striker above impact point

h_2 = Height of striker above impact point at zenith of rebound

L = Length of columns between base and longitudinal beam

M = Effective mass of structure or model

m = Mass of striker or pendulum

M_y = Bending moment at which yield stress is achieved somewhere in the cross section

M_u = Fully plastic bending moment

P_a = Axial column loading

P_e = Critical (Euler) column loading

R = Resistance of the model to horizontal deflection

$R(x)$ = Resistance function

σ = Material yield stress

$\bar{\sigma}$ = Average material yield stress

t = Time

v_1 = Velocity of striker at instant of impact

v_2 = Velocity of striker at instant after impact

V = Initial velocity of structure or model resulting from applied impulse

W = Actual weight of the longitudinal beam and its connections

w = Actual weight of the column length L

x = Any horizontal deflection

x_{dyn} = Maximum horizontal deflection of the model

x_{st} = Horizontal final static rest deflection of the model after impact

x_u = Horizontal deflection at which the fully plastic bending moment M_u is attained in idealized theory

1. Introduction

There has been considerable interest in analyzing the dynamic behavior of structures when subjected to very intense loads for very short periods of time. Sources of such loadings include blast from nuclear as well as chemical weapons, projectiles, slamming of ships in a seaway, wave action and earthquakes. If the loadings are not too severe the structure can be assumed to behave linearly and the wealth of procedures using linear vibration theory may be applied. However if the loadings are sufficient to cause damage it is not reasonable to assume linear behavior. Hence, in a large proportion of the cases of practical interest it becomes necessary to study inelastic behavior.

During recent years there has been an increasing use of and confidence in the techniques of "limit design" and "plastic analysis" for the treatment of structures built of ductile engineering materials. This type of analysis is quite realistic in predicting the static behavior of steel structures as tests have shown [1]*. In theory, plastic analysis of such structures could be applied to situations of dynamic loading. This has commonly been done by arbitrarily applying intensification factors to the static loads.

Many studies have shown that dynamic behavior is

*Square brackets refer to bibliography.

complicated by a phenomenon in which the yield point is increased over that obtained by standard (slow strain rate) tests. This effect of increase in dynamic yield strength is also related to a delay in the initiation of plastic behavior under dynamic loading. Dynamic yield strength is also dependent on the strain rate to which the material is subjected.

Thus it appeared to be of interest to study experimentally the actual dynamic behavior of a simple structure which is easily analyzable by simple theory. One can then determine whether or not simple theory provides safe predictions of actual behavior, and what influence the supported mass of the structure has on the behavior of the structure during its deformation. The structure chosen for this investigation is an idealized portal frame, the horizontal member of which is conceived of as being infinitely rigid and comprising most of the mass of the structure.

Some theoretical investigations in this field of dynamic loading of structures have been made by Brooks and Newmark [2]. They provide a number of charts to predict the behavior of idealized structures. Their charts are not applicable to situations wherein the structures were subjected to very short time duration loadings. Furthermore, ideal linear materials were assumed.

Another investigator, Tanaka [3] made a classical analysis of a square portal frame under horizontal impact.

He determined the effect of the mass of an elasto-plastic longitudinal beam on the plastic deformation. Many simplifying assumptions were made.

It appeared, therefore, that an opportunity existed for making an experimental study of a dynamically loaded simple portal frame. This study is to be discussed in the sections that follow.

2. Background For Investigation

In order to make an experimental study of a portal frame subjected to a rapidly applied lateral load, it was necessary to use models since full scale tests, such as those conducted at Lehigh University [1], were not feasible. A model was conceived which would permit testing of the essential portions of the elementary analysis.

The upper beam of the model was deliberately chosen to be many times more rigid than the supporting columns so as to simplify the study and to allow reuse of part of the model. This meant that the only expendable portions in each model test were the columns.

The supporting columns were chosen to be rectangular in cross-section with one side of the section many times greater than the other. This gives comparatively great transverse rigidity to the model in relation to the lateral load applied. This rigidity tends to ensure that motions caused by the lateral load are in the "plane" of the model; that is, in the general direction of the applied load. (See Fig. 1). It was decided that longitudinal loading in the plane of the frame could be most easily obtained by using a pendular mass. The mass is swung on long wires from known heights to produce impacts of varying momentum on the model.

There were two possible ways of simplifying the loading on the model so that a simple analysis could be made of what actually occurred. One method would be to have the pendular load act on the structure over such a short period

of time that all the momentum transfer occurred before the model had a chance to move appreciably. The other method was to cause the pendulum to strike the model and then attach itself to the model, continuing as a part of the model. Since the second method appeared to offer substantially greater mechanical difficulties than the first, it was not considered further. All loadings were accomplished by a pendulum which had only a very brief contact with the model.

A study of the theoretical considerations involved in this thesis shows that the basic divisions of the problem are: the motion of the model, the resisting force on the model, and the exchange of momentum between the striker and the model.

The motion of the model will be considered first. A model of mass M, which has a zero horizontal displacement ($x=0$) at the initial time ($t=0$), is initially at rest so that $V_o=0$. Then the mass M of the model, instantaneously is given an initial velocity V. (For a discussion of the effective mass of the model see Appendix I). In displacing horizontally, the mass encounters a resistance R which is dependent on the displacement, x. The resistance R, does negative work on the mass M, slowing it down; ultimately bringing it to rest. It can then be said, from Newton's second law of motion, that

$$R(x) = -M \frac{d^2x}{dt^2} \qquad \text{Eq. A}$$

R exists for all deflections of the model. The force

exerted by the mass on the structure does positive work, thereby distorting the structure. The extreme distortion of the structure occurs at the extreme displacement of the mass. This is when M first comes to rest. (This dynamic displacement is termed x_{dyn}). Then the work done by the restraining force R, on the mass M, in bringing it to rest must be equal to the initial kinetic energy of the mass due to its initial velocity, V. Thus,

$$\int_0^{x_{dyn}} R(x)\,dx = \tfrac{1}{2} MV^2 \qquad \text{Eq. B}$$

Most of the work done by the mass will have been expended in plastically deforming the structure; the rest is in elastic strain energy. After the extreme deflection x_{dyn} is reached the elastic work stored in the structure will cause the model to spring back to another position, x_{st}. The distance $x_{dyn} - x_{st}$ represents the elastic recovery of the model The resistance function R(x) is the resistance R which the structure gives to the mass M as it moves through a horizontal deflection x (see Eq. A). The following is a development of the resistance function for a portal frame using plastic limit analysis methods.

It is assumed that the steel of the portal frame's columns will behave elastically up to the actual yield point of the steel and then flow plastically without increase in stress when strained further. When the cross-sections of the columns have achieved the fully plastic bending moment

(M_u), which is greater than the bending moment at first yielding (M_y), a plastic hinge is formed. An ideal plastic hinge is a point hinge which undergoes rotation of any magnitude while the bending moment remains constant.

For a material of rectangular cross-section subjected to bending about an axis parallel to the width and through the center of the section:

$$M_y = \frac{(\text{width})\ (\text{height})^2}{6} \quad \text{and} \quad M_u = \frac{(\text{width})\ (\text{height})^2}{4}$$

These consider bending stresses only. Axial loads if significant would require a moment reduction factor. The axial loads encountered in this study were small enough so as not to require use of such a factor on M_y or M_u.

If a sufficiently large lateral force F is applied along the axis of the rigid longitudinal member, there will be four locations at which the bending moment will be a maximum simultaneously. These four are located at A, B, C and D, Fig. (2a).

When being deflected in the elastic region of the columns, the structure will appear as in Fig. (2b). A force analysis of half of one column during purely elastic deflection will show, as in Fig. (2c), that

$$R = \frac{24\ EI\ x}{L^3}\left(1 - \frac{P_a}{P_e}\right) \qquad \text{(elastic deflection)} \qquad \text{Eq. C}$$

Where P_a is the axial load on the column $\left(P_a = \frac{W+w}{2}\right)$ and P_e is the critical Euler column loading as determined by

$$P_e = \frac{\pi^2 EI}{L^2}$$

Thus Eq. C is the resistance function of the structure in the elastic region until M_u is achieved.

The vertical elements of the structure are beam columns. The effect of the lateral loads is intensified by the presence of axial compressive loads. While it would be possible to carry out an exact evaluation of the intensifying effect, the theory of columns indicates that this simple intensifying factor is sufficiently accurate almost up to ratios of $P_a/P_e = 1$ [4]. That is, to loads almost equal to the critical load for the column.

Once M_u is achieved, the idealized structure develops four plastic hinges at A, B, C and D (Fig. 2d). A force analysis of one column of the symmetric model Fig. 2e which accounts for the weight of the column and the support beam gives:

$$\frac{W}{2}x + w\frac{x}{2} + \frac{R}{2}\sqrt{L^2 - x^2} = 2M_u$$

For the entire model (2 columns):

$$R = \frac{4M_u - (W+w)x}{\sqrt{L^2 - x^2}} \qquad \text{(plastic deflection)} \quad \text{Eq. D}$$

This equation could be simplified for small deflections by neglecting x^2 in comparison with L^2. However, large deflections are encountered in this study and such simplifications would introduce needless errors.

Equation C represents the resistance function of the structural model in the elastic region. Equation C applies up to x_u, the horizontal deflection at which the fully plastic bending moment, M_u is attained. Equation D applies after x_u and represents the resistance function of the model in the plastic region.

This is graphically shown in Fig. 3. The curve in Fig. 3 has a sharp corner as plastic theory would indicate [1]. There is a slight idealization in this curve since elastic action and plastic action which are occurring simultaneously before x_u, actually cause a rounding of the corner. The error involved in this idealization is slight. The resistance function then is:

$$R(x) = \frac{24EIx}{L^3}\left(1 - \frac{P_a}{P_e}\right) \qquad 0 \leq x \leq x_u$$

$$= \frac{4M_u - (W+w)x}{\sqrt{L^2 - x^2}} \qquad x_u \leq x \qquad \text{Eq. E}$$

x_u is the horizontal deflection at which the fully plastic bending moment M_u, is achieved. It represents that value of x which simultaneously satisfies both parts of Eq. E.

It is most easily obtained by equating the right hand sides of Eqs. C and D and solving for x_u by trial and error. Thus:

$$\int_0^{x_{dyn}} R(x)\,dx = \frac{1}{2} MV^2 = \frac{12\,EI}{L^3}\left(1 - \frac{P_a}{P_e}\right) x_{dyn}^{\;2} \qquad \text{for } 0 \le x_{dyn} \le x_u$$

Eq. F

and:

$$\int_0^{x_{dyn}} R(x)\,dx = \frac{1}{2} MV^2 = 4M_u \sin^{-1}\frac{x_{dyn}}{L} - \left(L - \sqrt{L^2 - x_{dyn}^2}\right)(W + w)$$
$$- \left[4M_u \sin^{-1}\frac{x_u}{L} - \left(L - \sqrt{L^2 - x_u^2}\right)(W + w) - \frac{12\,EI}{L^3}\left(1 - \frac{P_a}{P_e}\right) x_u^{\;2}\right]$$
$$\text{for } x_u \le x_{dyn}$$

Eq. G

The square bracketed portion of Eq. G is constant and permits simplicity in repeated computations.

Next, examine the exchange of momentum between the pendulum and the structural model. To produce motion of the mass M such that it almost instantaneously achieves an initial velocity V, we can apply a very large load of very short time duration. If such a loading, acting on the mass of the structure, is ended before the mass moves and the structure begins to develop resistance, the mass of the structure M can be considered to have effectively developed an initial velocity V. The initial kinetic energy of the

mass M can then be said to be due to the impulsive loading. Hence,

$$\frac{1}{2} M V^2 = \frac{\left(\int_0^t F\,dt \right)^2}{2M} \qquad \text{Eq. H}$$

The impulse comes from an exchange of momentum between the pendulum (or striker) and the mass of the structure. Use of the principle of conservation of momentum evades the problem of attempting to evaluate the significant energy loss during impact.

The principle of conservation of momentum states that

$$(mv)_1 + (MV)_1 = (mv)_2 + (MV)_2 \qquad \text{Eq. I}$$

If we recognize that the structure is at rest initially, with $V_1 = 0$, hence $(MV)_1 = 0$; Eq. I simplifies to

$$(mv)_1 = (mv)_2 + MV \qquad \text{Eq. J}$$

Solving Eq. J for MV it can be said that the change of momentum of the model, as a result of being struck by the pendular mass, is equal to the change in momentum of the pendular mass

$$MV = m(v_1 - v_2)$$

The change in momentum of the structure is due to the applied impulse, hence,

$$MV = \int_0^t F\,dt$$

from which Eq H is readily obtained.

Thus, the change in momentum of the pendular mass (the striker) can then give the initial kinetic energy of the mass of the structure

$$\frac{[m(v_1 - v_2)]}{2M} = \frac{1}{2}MV^2$$

Eq. K

In summation, then, it is seen from Eq. K that by knowing the change in momentum of the striker, one can infer the initial kinetic energy of the mass of the structure M. With this kinetic energy and the use of Eq. F one can then predict the maximum deflection x_{dyn} an ideal portal frame would undergo when dynamically loaded.

The beauty of this analysis is that, regardless of the deformations to the striker and the model during time of impact, one can determine the kinetic energy of the structure after impact.

3. Experimental Equipment

There are three main areas to be discussed concerning the equipment used in conducting the experiment. These are the structural model, the striker and the measurements of what happened.

Structural Model

As shown in the sketch of the model (Fig. 4), the top beam is a solid bar of hot rolled steel measuring 2" x 2" x 17-3/4". The ends were tapped to receive 5/16" N.C. socket head cap screws which attached the columns to the beam.

The beam endured four modifications which allowed the "mass of the structure" parameter to be varied. The first beam (MK-I) was the plain steel bar described above. The second beam (MK-II) had two half-inch plates welded to the top of the bar, Fig. 4b. Two lead bricks were placed on the beam and firmly wedged into place to prevent horizontal motion. The bricks were held down by a top bar to restrain them from moving vertically. The third beam (MK-III) still used the same bar as in the MK-I model, but the lead bricks were replaced by solid billets of steel which were welded to the 2" x 2" original bar, Fig. 4c. The billets were spaced about a point approximately in line with line of impact. The fourth beam (MK-IV) saw a removal of some of the steel welded to the MK-III. This was done to duplicate the mass of MK-II so that verification could be made of the data obtained in the MK-II test series. The total weight (W) of the longitudinal beam, fastening plates and

connections was MK-I, 20.92 lb.; MK-II, 78.46 lb.: MK-III 120.0 lb.; and MK-IV, 78.48 lb.

The columns were made from hot rolled mild steel strip conforming to ASTM A303-58T standards. The average cross section of the strip showed a width of 1.97" and a thickness (after scale removal) of 0.120". The width varied from 1.96" to 1.98", but the number of samples which varied from the average was small. The thickness varied from .118" to .122" but the great majority of the pieces sampled were very close to 0.120" in thickness.

This steel was chosen because it closely resembles structural grade steels. It was certified to be all from the same heat. Its average static yield as determined by a standard tensile test specimen and A-1 electrical resistance strain gages connected to a B-L-H Model N strain indicator was 41,100 psi. Tensile tests were conducted on a Riehle 60,000 lb. Universal Testing Machine. Further details on composition of steel and its physical properties are found in Appendix II.

The steel was cut into 22" lengths from 10 foot long strips sent from the mill. A pattern was used to drill the holes through which the columns were bolted to the beam and the base. Two inches at each end of the strip was reserved for the mounting area. This left almost exactly 18" for the column length. At every connection of the columns to either the base or the beam, a cover plate (or fastening piece) was used between the cap screw heads and the columns.

This allowed the fastening pressure to be fairly uniform over the end of the column.

All edges on the beam, base, or fastening pieces about which a plastic hinge was expected to form, were machined to a right angle.

The fastening piece at the struck end was fashioned from a ¼" plate. Centrally mounted on it by a press fit was a hardened steel button, one inch in diameter and ¼" thick. This button was the target for the pendular mass. Three quenched and tempered buttons in all were needed, since two were broken by the force of impact.

The after fastening piece had a phonograph needle or, later, a canvas sewing needle, imbedded in it. The needle scratched a paraffin block placed in way of the expected motion of the model Fig. 5. The needles and their mounts were sufficiently rigid to have negligible deflection. There was no apparent transverse motion of the moving model. The model was bolted to a support base through its columns. The base was constructed of a welded 1" plate frame bolted to a ¼" plate. This bottom plate sat in a steel lined groove in the concrete floor which effectively prevented motion in the direction of the motion of the pendular mass. By addition of assorted steel billets in and on the frame base, the mass of the base approached some 500 lbs. This was enough to prevent its motion during impact.

The rough proportions of the model were randomly selected to approximate existing structures. The actual

dimensions were arrived at by preliminary calculations of forces needed, materials available and space required for the tests.

Striker

The striking force on the model was provided by a mass swung as a pendulum. The support for the pendulum was provided by a conveniently located overhead monorail (part of a chain hoist system). Rigidly bolted to this monorail was a heavy 4" by 4" angle iron framework. Bolted to the framework were small ball bearings spaced three feet apart. These bearings served as the pivots for the pendulum. To provide a means of attaching the pendulum wires to the bearings, thimble eyes were fitted over the bearings. The support wires were then passed over and tied to the thimbles.

The pendular mass was thus given bi-filar suspension using .024" diameter music wire. Originally galvanized guy wire was used but it proved to be too stiff and was replaced with the music wire. The connecting link between the mass and the wire was a short piece of braided nylon line. This permitted easier length adjustments than the wire and also electrically insulated the model from the music wire.

The first striker mass (No. 1) was a cylindrical steel bar supported by four wires (2 bi-filar suspension sets). Fine adjustments in truing up the striker to the impact button were accomplished by four fine thread turnbuckles, one on each wire. This system gave good impact but only fair rebound owing to the oscillations that the

striker underwent after impact. The turnbuckles are believed to have been instrumental in causing the oscillations The striker padeyes and hauling tail weighed 6.38 lbs. The effective striking mass including one half the mass of the turnbuckles (owing to their proximity to the striker was 6.57 lbs.

The second striker mass (No. 2) was a solid sphere on a shaft and with necessary bolts and washers weighed 7.11 lbs. This second striker, much like a cannon ball on a shaft, was machined from a solid billet of cold rolled steel. The cannon ball was suspended by one bifilar set. The connecting link to the music wire was braided nylon line which allowed easy adjustments by varying its length. The cannon ball was 3½" in diameter.

The third pendular mass (No. 3) was a solid steel disk with a rounded edge, on a steel shaft (much like a mill-stone on an axle). It was necessary to go to a larger pendular mass to strike the heavier MK-II, MK-III, and MK-IV models. Material for another, larger cannon ball was not readily available, but that for a disk was. The mill-stone, 7" in diameter and 2¼" thick, with a shaft 1" in diameter and 5" long, along with bolts and washers, weighed 22.9 lbs. The millstone employed a bifilar suspension with nylon connecting links, as did the previous strikers.

The fourth pendular mass (No. 4) was the millstone (No. 3) with the disk machined down to a 6" diameter. All other dimensions were the same. Its weight including bolts

and washers was 17.22 lbs.

Both the cannon ball and the millstone gave a very good account of themselves in the tests. Their impacts were sharp and the rebounds were clean without noticeable oscillations. It is important that the proper ratio of mass of striker and mass of model be achieved. This is to prevent interference between the striker and the model after initial impact. Such interference occurred in the low energy tests of the MK-II model and the No. 3 striker making it difficult to obtain accurate rebound data.

Strikers No. 2, 3, and 4, unlike the first striker, had their mass centers close to the impact point. This is desirable. The spherical striker and close imitations are desirable. Such shapes eliminate unwanted dynamic oscillations caused by release conditions, suspension systems, and off-center impacts. The shorter the striking body is, the better the likelihood of a short impulse time.

The back of each striker had a light wire harness onto which a releasing trigger was attached (Fig. 6a). From a hand winch and passed over two pulleys attached to the overhead monorail, a cotton braided line was attached to the releasing trigger (Fig. 6b). By cranking up the hand winch the pendular mass could be raised to any position.

The two pulleys were positioned on the rail so as to minimize introducing an initial tension load in the support wires (Fig. 6c). The intent of this was to allow the mass to swing freely in a circular arc upon release. In the

very first tests using the first striker, one of the pulleys was located so as to introduce a high initial tension in the support wires. Upon release of the striker, the tension was suddenly reduced. This caused an oscillation in the striker system in its downward flight. This condition was corrected in all later tests.

Measurements

To investigate if the conditions of short impulse time were met, measurements of the striker contact time were made. A Hewlett-Packard Model 522B Electronic Counter was employed. This device was able to measure and record time divisions as small as ten microseconds, with an accuracy of 20 microseconds on this application. Certainly this was good enough for the purpose.

The circuit employing the counter is shown in Fig. 7. A 45 volt dry cell in series with the counter was connected from its cathode through a long light wire to the striking mass. The ground side of the counter was then connected to the structural model close to the impact button. To insure good contact a silver conducting paint was applied to both sides of the impact fastening piece. The circuit was closed when the striker hit the impact button of the model. It was opened when the striker and model separated. The elapsed time of contact was displayed on the counter. Since the circuit outside of the machine involved only small resistive elements no circuit corrections for transient delays were necessary. The times measured

were almost always appreciably less than one millisecond.

Measurement of the rebound of the striker was at first a vexing problem. Using a time exposure, a photograph of the rebound against a background grid was made with a Polaroid-Land camera. The time exposure would show the zenith of the rebound. Picture quality was quite variable. Since one had to depend on one's eye to spot the rebound to check the reliability of the photograph, it was felt that one might just as well abandon taking the photographs, for a more foolproof technique.

Now the striker was caught by hand at the zenith of its rebound and its vertical distance measured from the floor. The rebound trajectory passed close and parallel to a calibrated grid against which it was easy to spot the vertical distance. While this was a fairly crude method it gave satisfactory results.

All vertical distances were measured from the floor as a base. It was found that the cement floor in Halligan Hall under the swing of the striker was quite flat. It was level to the horizon within one sixteenth of a degree. Flatness was checked by stretching a wire and measuring the vertical distance. Compensating for the catenary of the wire the only significant variation from the flatness was at the trench where the models were installed. This variation was a drop of one eighth of an inch and was compensated for in the conduct of the experiment.

The height of the striker before release was measured

by passing a small waxed line around the axle of the striker and letting it hang to the floor. The waxed line was removed before release of the pendulum and measured. The waxed line was used because it resisted stretching in tension quite nicely.

The other important measurement was of the motion of the impacted model. As mentioned previously, this was done by having a phonograph needle (and later a canvas sewing needle), imbedded in the model, make a record of its motion on a paraffin block. This block was cast onto a wood backing plate which had two carriage bolts driven into it. The block was then mounted on a wooden stand with slots in which the bolts could be slid up and down. This arrangement allowed using the same block for many scratches and permitting moving the block to the best position for receiving the scratch. Since the paraffin offered resistance, though very little, to the motion of the model, it was desirable to record only the last three inches or so of travel, in order to reduce this resistance. Accordingly, the paraffin block was located so that the scratch recorded was no longer than needed to obtain the maximum dynamic deflection of the model.

Three measurements of location of the model were made: (1) before impact; (2) at point of maximum deflection; and (3) at rest following impact. The measurements were made vertically and horizontally from the needle to a reference base. Levelness of the longitudinal beam of the

model was checked before and after impact. Deviations from the horizontal were not observed in any test except those which led to complete collapse where the model fell to the floor.

The resistance of the paraffin blocks was determined by making scratches of known depths with a needle moving under known calibrated forces. A single pulley arrangement was employed with the needle and its mount hanging on one side and calibrated weights on the other. Though the resistance was small, it was taken into account in the energy relations discussed in Part 5 of this thesis.

4. Experimental Procedure

The sequence of events and measurements in a typical experimental run in which a model was deformed plastically is as follows:

First, assemble the model in position on the test base. This is most easily accomplished by bolting the columns onto the base first and aligning them to the vertical. The beam and its fastening pieces are then tightly bolted to the columns. The MK-I and -II beams were positioned by hand and the MK-III and -IV beams were positioned with the aid of a chain hoist. The ground wire to the electronic counter is attached to the fastening piece with the impact button on it.

Next the wax scratch block is aligned with the expected path of motion of the needle on the model. The paraffin block is oriented so as to present an unscratched surface to the needle. The scratch block stand is weighted down to prevent its accidental movement.

Now, determine the initial location of the model needle from horizontal and vertical references. Then, check the electronic counter circuit to see if it is functioning properly. The counter should have been warmed up by running it for some thirty minutes.

Attach the waxed measuring line to the striker. Hook the releasing trigger onto the hauling harness of the striker. Crank up on the hand winch until the striker is at the desired initial height. Remove the waxed measuring

line from the striker and record the initial height.

Pass the trigger release line over a pulley on the cement floor, to the location of the experimenter. The experimenter places himself about three feet behind the struck side of the model after clearing the test area. The release line is slowly pulled until the trigger flies open and the striker is on its flight.

The striker hits the impact button and rebounds in the direction from which it came. The experimenter seizes the striker at its position of highest rebound. He quickly makes a measurement of the striker's vertical height. The striker is then slowly lowered to a rest position.

The dynamic motion of the model is recorded on the paraffin block. The position of the point of maximum scratch (maximum dynamic needle deflection) is measured and recorded with respect to the vertical and horizontal references. The rest position of the needle is recorded in the same manner.

The time of contact between the striker and the button (impulse time period) is read directly from the electronic counter. Since the impact generally involves clean separation of the striker from the model, the counter can be read at one's leisure.

5. Discussion Of Results

All data collected, both raw and reduced are tabulated in Appendix III.

Appendix IV contains a tabulation of computed data as obtained from the deflection theory developed earlier

Figures 8, 9 and 10 display the reduced experimental data against a background of curves obtained from large deflection theory. An important parameter in the theory is the weight of the longitudinal beam, its connections and the columns. Figures 8, 9 and 10 differ from each other in that each one is for a separate "weight of structure."

Another parameter is the yield stress ($\overline{\sigma}$) of the column material. Three yield stresses were chosen to obtain theoretical deflection curves for each of Figs. 8, 9 and 10. These yield stresses are: 41,100 psi (the average test yield stress), 50,000 psi, and 60,000 psi.

Figure 8 shows the theoretical curves for the MK-I structural model. The experimental data points for both the No. 1 and No. 2 striker are shown thereon. Although the first one was abandoned as a striker, the experimental data fit in nicely with that obtained by the No. 2 striker.

<u>Variations and Error</u>

It is apparent from Fig. 8 that the data obtained are reasonably consistent and reproducible. Any variations from a presumed mean value of data are likely to be due to slight variations in the physical strengths of the columns and their end conditions from one test model to the next.

Inaccurate measurement of the motion of the model as recorded on the wax scratch block was one obvious source of error. The measurements could only be made reasonably to the nearest 1/64" and sometimes to only 1/32".

Another error was introduced by inaccuracies in measurements of the rebound height of the striker. The closest that this could reasonably be measured to was ± ¼". Inaccurate measurement of this order on large momentum tests would lead to an error of less than one per cent in the total momentum transferred.

Also, some error resulted from off-center impacts of striker against the model, causing the latter to depart slightly from a plane circular path. It is not possible to estimate the size of such errors.

Corrections were made for aerodynamic resistance, strain hardening, and resistance of the wax block to being scratched by the model's needle. These will be discussed more fully later.

Test Series (MK-I)

The experimental data point labeled "19½" is displaced to the right of a presumed mean value by a goodly amount. This model was actually model Serial 19 which, after undergoing plastic deformation, was straightened out and used again as "19½". The compressive and tensile stresses introduced into the columns by cold forming (straightening) resulted in reducing their yield strengths during the subsequent test. A greater deflection of the impacted model

resulted.

Since very slight variations in material test conditions or in recording the rebound of the striker appreciably affect the results of the low momentum tests not too much significance is to be attached to that portion of the data; except perhaps, that it closely follows simple theory.

The high and medium momentum tests do show a proclivity to deviate from simple large deflection theory. The implication from Fig. 8 is that the columns are behaving as if they were stronger (ie. higher yield stress). Therefore, it behooves one to scrutinize these points more closely.

It was assumed, in the calculation of the momentum of the striker, that the velocity at impact was the same as that obtained by a freely falling object (in a vacuum) from the initial height of the striker (h_1). This is not wholly true. There is resistance of the pendulum's pivots and aerodynamic drag of the striker in flight. The total of such resistance as found from actual free swinging tests was not insignificant. For the maximum initial height (h_1) that the No. 2 striker was pulled to, the work done by the drag was around 27 in-lb. This loss was found to be appreciably less for lower velocity tests, as theory would predict. The first striker had only slightly lower resistance.

A correction for drag, on Fig. 8 would cause the experimental data points to be displaced downward an appropriate

amount.

Another correction to the experimental data is that for strain hardening of the steel columns when grossly deformed. Fortunately the plastic deformation of the columns took place only in the vicinity of the theoretically predicted hinges. This is precisely as predicted by plastic theory. Since the hinges did not and could not become ideal point hinges, they spread out over a finite distance. The greater the horizontal deflection the greater the curvature of the metal at the hinge.

Such curvature due to bending of the columns caused a reduction in the straight-line length of the column from the mounting base to the longitudinal beam. On one model which had deflected horizontally some 10½" this apparent "shortening" of the columns was measured to be 1/8". This resulted in a loss of about 1/16" of potential horizontal deflection from what an ideal point hinge would permit.

Correction for this strain hardening would cause the comparison curves to be shifted to the left. For ease it would be best to apply a correction to the experimental data point and shift it to the right.

A third correction which should be considered is the resistance offered to the model by the wax the phonograph needle is scratching. This resistance depended on the depth and length of the scratch as well as the speed with which the scratch was being made.

Rough measurements were made of the paraffin's

resistance as to depth of scratch. The average resistance for a 1/8" deep scratch was measured to be 2.5 lb. Resistance increased for deeper scratches, decreased for shallower scratches. The average length of the scratch was approximately 4" long. It was noted that the paraffin's resistance dropped appreciably to perhaps half the static value, once the needle began moving. On this basis one could say that average work done on the wax during a 1/8" deep cut 4" long was around 5 in.-lb.

The experimental data points should be lowered on Fig. 8 to correct for the resistance offered by the paraffin scratch block.

These three determinable effects (strain hardening, resistance of wax to model needle, drag resistance of the striker) cause a net shift of the data points down and to the right. Lower momentum data points were not corrected for drag, etc., since such correction for these points was negligible. These approximate corrections are shown in Fig. 8. The corrected points now closely fall in line with the theoretical deflection curve plotted for $\bar{\sigma}$ = 50,000 psi. This particularly holds for the high momentum points. As momentum is decreased, (i.e., V is decreased) $\bar{\sigma}$ approaches the static test value $\bar{\sigma}$ = 41,100 psi.

With increasing initial velocity of the structural model (from high momentum tests) the loading of the columns by bending is quite rapid, causing a high strain rate. Steel usually displays a delayed yield effect under high

strain rates [5]. Samples of the same material used in this experiment did display such a delayed yield phenomenon in tests conducted by the David Taylor Model Basin Appendix V. Therefore it is suggested that a good portion of this seeming increase in strength of the columns may be due to this delayed yield effect.

While data from the tests conducted at the David Taylor Model Basin can be arranged to show the time dependent, increased yield points, not enough tests were conducted to get a complete picture of the actual delayed yield of this batch of steel.

In order for a MK-I structural model to reach a deflection of around 10" it would require an initial velocity (V) of around 150 in./sec. This model velocity decreases to zero probably following some second or third order parabolic rule. Accordingly the plastic deformation of the model may begin when the model's velocity is in the neighborhood of say 140 in./sec. The horizontal distance the model would move before such yielding occurs would be about 2.3". It can be said that the time elapsed before yielding began would be of the order of 2.3 in./145 in./sec., or around 15-20 milliseconds. Reference to Fig. 11 shows that a time to yield stress of this order would give a yield stress of around 57,000 psi. This is some 14% above that guessed at from Fig. 8. However, it is quite likely that the delayed yield obtained in tension as in the Model Basin tests, is not directly comparable

to a delayed yield obtained by bending, as in the experimental tests of this thesis. While research has been done on the delayed yield effect separately in tension and in compression, none has been conducted in axially loaded bending, which involves tension and compression simultaneously in one specimen.

The approximations in the foregoing discussion of delayed yield effects are gross. The test data and knowledge of yielding during high strain rates is not sufficient to warrant anything else.

Test Series MK-II, MK-IV

Figure 9 shows the experimental data points obtained from the MK-II and MK-IV structural models impacted by the No. 3 and No. 4 strikers, respectively. These data are displayed against theoretical curves obtained by large deflection theory for material yield stresses of $\bar{\sigma} = 41,100$ psi, 50,000 psi and 60,000 psi.

The MK-II and MK-IV data are shown together because their weight of structure (W) is virtually the same. The MK-II structural model, it will be recalled, used lead bricks to increase the weight of the structure. The MK-II longitudinal beam was composed of welded billets of steel.

Owing to the interference that existed between the impacted MK-II model and the No. 3 striker at the lower momentum tests, there was some doubt as to the reliability of these data. Then, as the data from the MK-III tests were assembled it became intuitively apparent that the

MK-II data, while consistent and reproducible, were of the wrong magnitude. The MK-IV tests confirmed this.

The major reason for the large deviation of the MK-II tests from the MK-IV tests was due to the lead in the model. The recrystallization temperature of the lead is below room temperature. Thus, lead may deform continuously and easily without strain hardening. The sudden, high acceleration imposed on the structure by the impulse caused the lead bricks to deform. The lead apparently continued to flow after the impulsive load was removed. While the striker remained in contact with the beam only a very small fraction of a second, it is likely that there existed a relative velocity between the center of mass of the lead and the center of mass of all else except the lead. This relative velocity meant that flow continued to occur for a short period after the striking impulse terminated. The lead, therefore absorbed some of the kinetic energy that the structural model had, preventing the plastic hinges from getting all of it. Hence, the horizontal defections of the MK-II model were much decreased. The deformation of the lead bricks was visible to the eye.

The MK-IV tests were conducted as a check on the MK-II series with only a limited number of specimens available. Nevertheless, sufficient data were collected to show the deviation of the MK-II tests as well as the approximate behavior of a model with a weight of structure between that of the MK-I series and the MK-III series.

The same general corrections discussed earlier for the MK-I series apply also to these tests. The resistance of the wax to being scratched and the strain hardening was of the same order as before. The total drag of the No. 4 striker, however was slightly different. The work done by drag was measured to be around 15 in-lb. for a maximum initial drop height (h_1) of 114".

These approximate corrections are reflected in corrected high momentum data points in Fig. 9 for the MK-IV series only. It will be observed that a mean curve if projected through these corrected points, corresponds closely to a large deflection theory curve for $\bar{\sigma} = 49{,}000$ psi.

With an initial velocity of the model (V) of around 75 in./sec. it takes about 30 milliseconds before the columns begin to yield. This would correspond to a delayed yield strength from Fig. 11 of about 56,000 psi. Since the presumed mean value in Fig. 9 is some 49,000 psi, the Fig. 11 value is approximately 12% higher.

<u>Test Series MK-III</u>

Figure 10 displays the experimental data points obtained by impacting the MK-III structural model with the No. 3 striker. The deflection curves in Fig. 10 were obtained by large deflection theory using material yield stresses of $\bar{\sigma} = 41{,}000$ psi, $\bar{\sigma} = 50{,}000$ psi and $\bar{\sigma} = 60{,}000$ psi.

The data obtained show good reproducibility. With little effort a median curve could easily be found to

represent the data. Instability, leading to complete collapse of the structure was achieved. Two of the data points, seemingly far from a median value, will be commented on.

Data point 34-2 is a retest of previously deformed model 34-1. Cold straightening the columns of 34-1 introduced residual stresses which effectively reduced the resistance to further bending. Accordingly, for a given loading the model deflected further. Data point 33-4 may have had its actual horizontal deflection incorrectly recorded.

Corrections as mentioned for Figs. 8 and 9 also apply here. The wax scratch resistance and the strain hardening are about the same for the MK-III tests as for the models discussed earlier. For a No. 3 striker swung from a maximum initial height of about 53", the drag loss was slightly less than 10 in-lb. When applied to the experimental data a new series of corrected data points on Fig. 10 would correspond closely to a large deflection theory curve for $\bar{\sigma}$ = 48,000 psi.

This value of $\bar{\sigma}$ = 48,000 psi, probably represents an increase in the yield strength of the material due to dynamic loading. To achieve its maximum deflection the structural model had an initial velocity of around 24 in/sec. The time interval before yielding in the columns began was of the order of 80 to 85 milliseconds. From Fig. 11, for such a time to start yielding, the yield strength is about

53,000 psi. This is an increase of around 10% over what was seemingly determined in the model tests.

6. Summary and Conclusions

Described in this thesis are the test equipment and the experimental procedure used to determine the behavior of a model of a simple portal frame under impulsive loading of very short time period. The loading is applied by a swinging pendular mass. The model was tested in the elastic region as well as the plastic region of deformation.

Experimental data as collected are compared with theoretical predictions made by simple deflection theory.

From these tests and their results it may be concluded that:

(1) the data obtained are consistent and reproducible.

(2) the equipment employed was of satisfactory design and worked well since the data it generated were satisfactory.

(3) the mass and the shape of the striker had no special effect on the behavior of the model after impact. Motion of the model depended entirely on the momentum transferred from the striker.

(4) the corrected data points in Fig. 8, 9 and 10 show that the structural model resisted deformation appreciably more than predicted by simple theory.

(5) the apparent increase in strength of the dynamically loaded model can most easily be explained by the strain hardening of the material through gross internal deformation (which is not easily measurable) and by

increase in the material's yield strength due to dynamic loading,

(6) it should be expected that the dynamic yield strength or the delayed yield phenomenon would depend on the initial velocity V. The work of others [5] and the dynamic yield tests in Appendix V agree with this. The results of the tests in this study show that there was a higher value for the dynamic yield strength in the tests wherein the initial model velocity V was greater (such as in the Mk-I) than in the tests where the model velocity V was the lowest (as in the Mk-III). However, there are not sufficient data to permit valid comparisons with similar correlations obtained by other means. The data collected in these tests indicates that the elevation of yield strengths is less than the elevation of yield strengths found in high strain rate tensile tests. The reliability of data from the low momentum tests was not sufficient to permit a similar comparison with $\overline{\sigma}$ in any one model series of tests,

(7) the deflection theory developed in Appendix IV and using the material's static yield stress, should be satisfactory for design purposes of portal frames of the same configurations as those of the model used in this study.

7. Suggestions for further work

The basic problem that the thesis was begun for was essentially completed. There are however, additional avenues of investigation which may be pursued. These include the study of multistory structures as well as structures different from portal frames. In addition, computer programs could be developed for mathematical models undergoing similar loading conditions.

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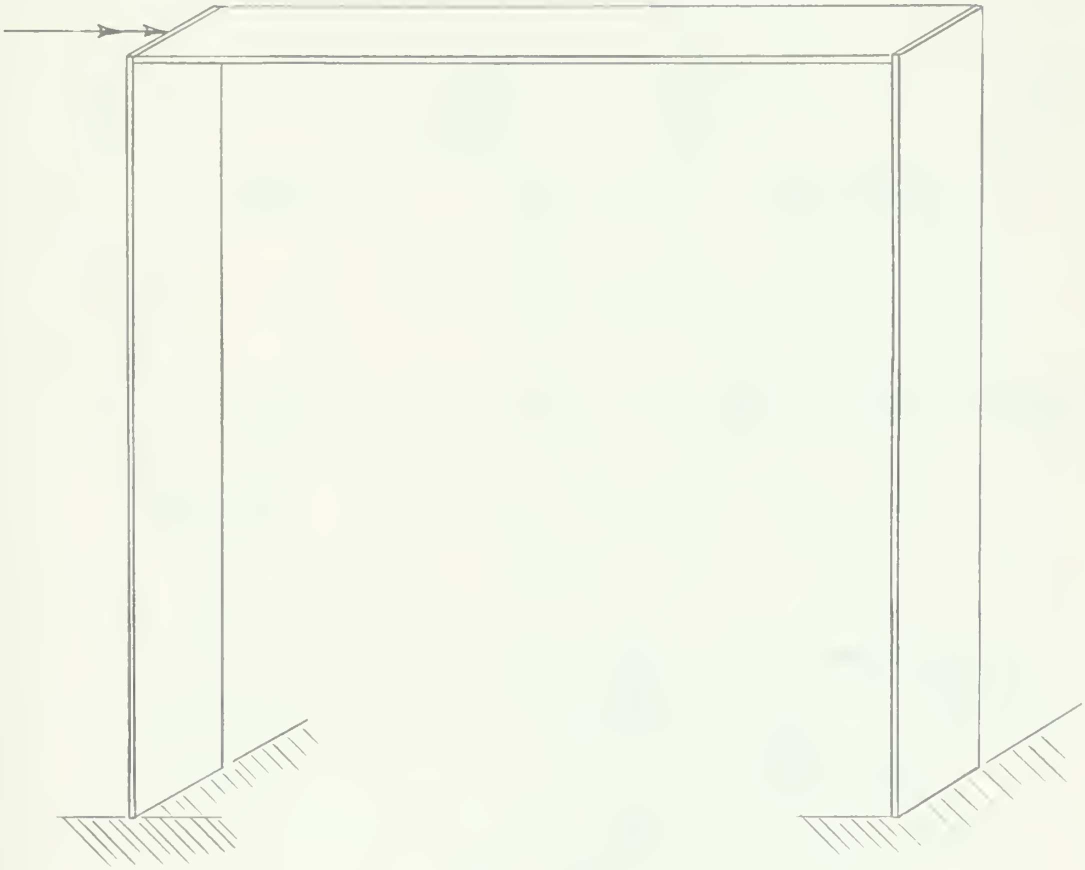
[3] "Plastic Deformations of a Square Portal Frame Under Horizontal Impact" by H. Tanaka. Technical Report No. 60, Division of Applied Mathematics Brown University, May 1960

[4] "Strength of Materials-Part I"- 3rd Edition p. 267 by S. Timoshenko. D. Van Nostrand Co., Inc., New York 1955

[5] "Delayed Yielding in Annealed Low Carbon Steel under Compression Impact" by E. E. Johnson, D. S. Wood and D. S. Clark, p. 755 ASTM Proceedings, Vol. 53, 1953



Idealized Portal Frame

Fig. 1

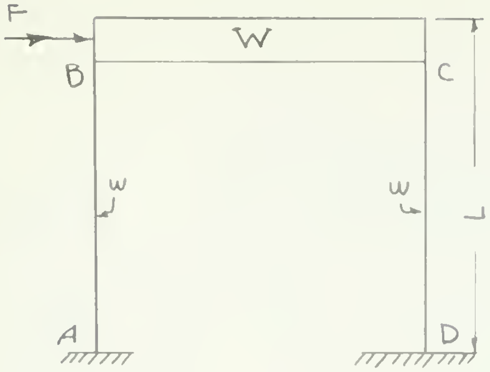


Fig 2a

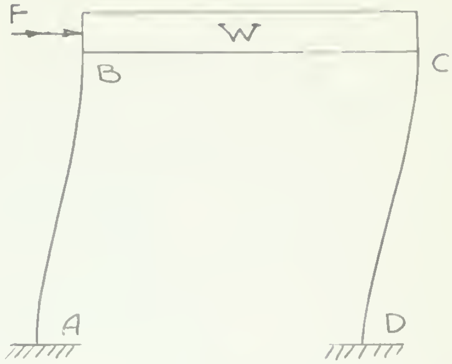


Fig 2b

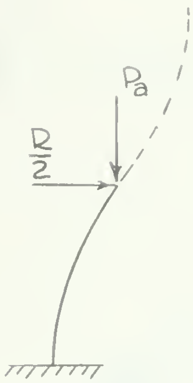


Fig. 2c

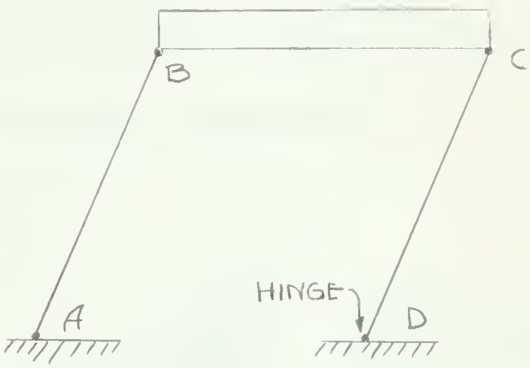


Fig 2d

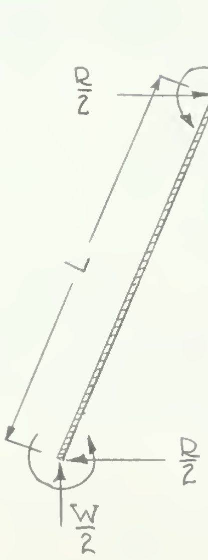
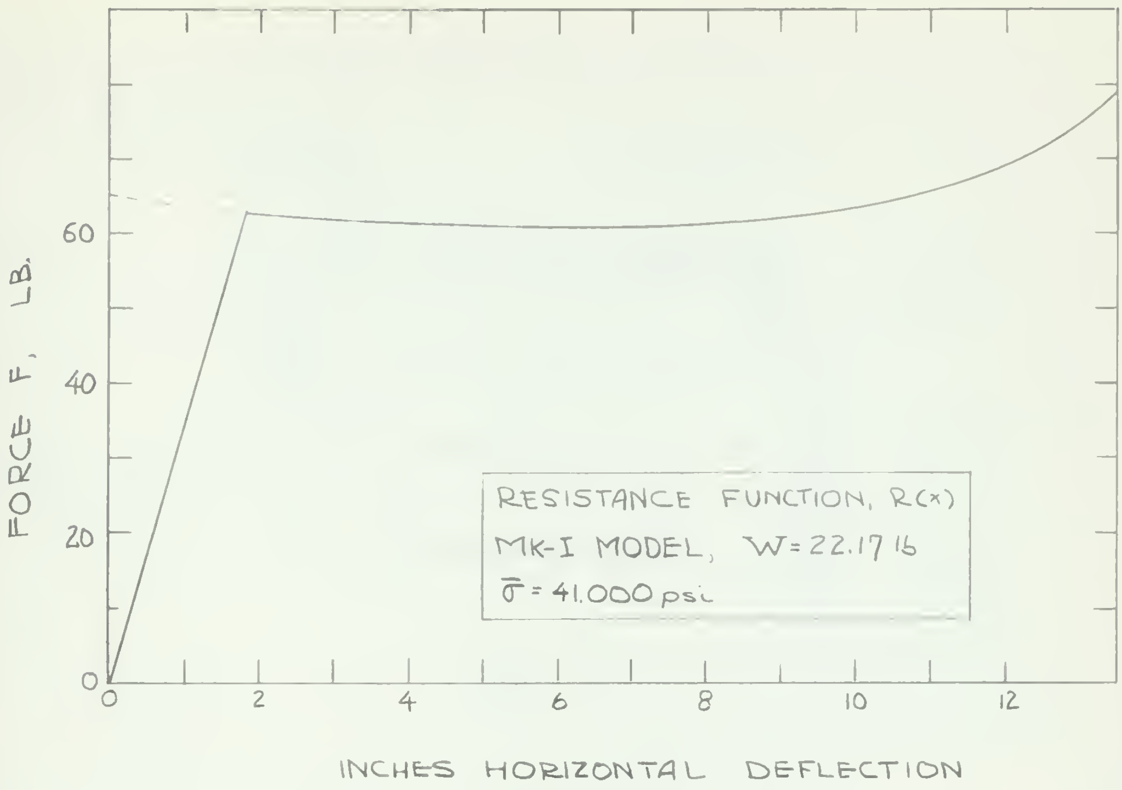
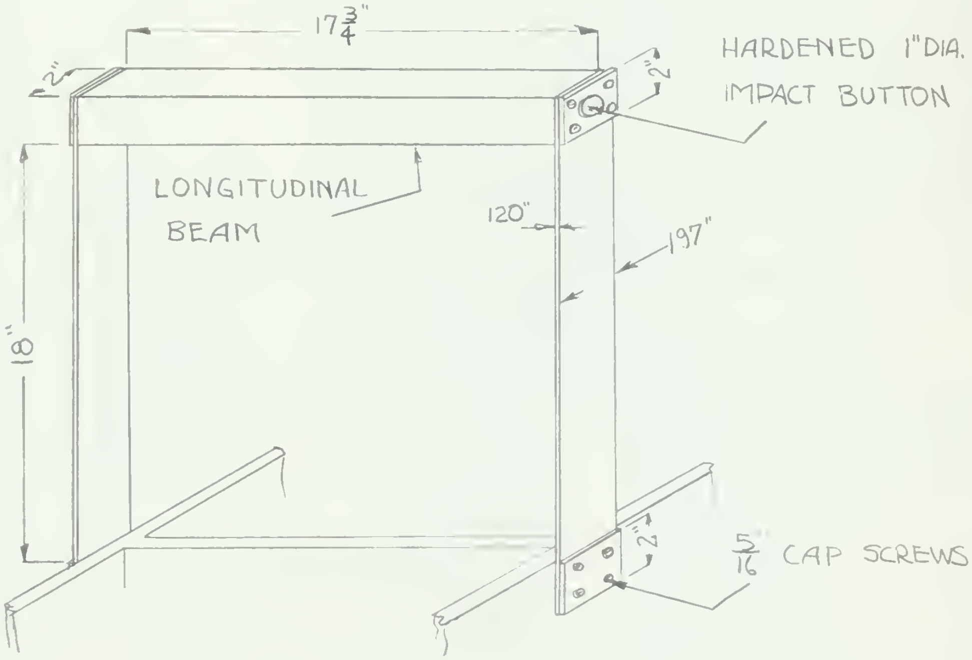


Fig. 2e



FORCE F, LB.
0
20
40
60
0
2
4
6
8
10
12
INCHES HORIZONTAL DEFLECTION
RESISTANCE FUNCTION, R(x)
MK-I MODEL, W=22.17 lb
$\bar{\sigma}$ = 41,000 psi

Fig. 3



$17\frac{3}{4}$"
2"
2"
HARDENED 1" DIA.
IMPACT BUTTON
LONGITUDINAL
BEAM
18"
.120"
.197"
2"
$\frac{5}{16}$" CAP SCREWS

Fig 4a

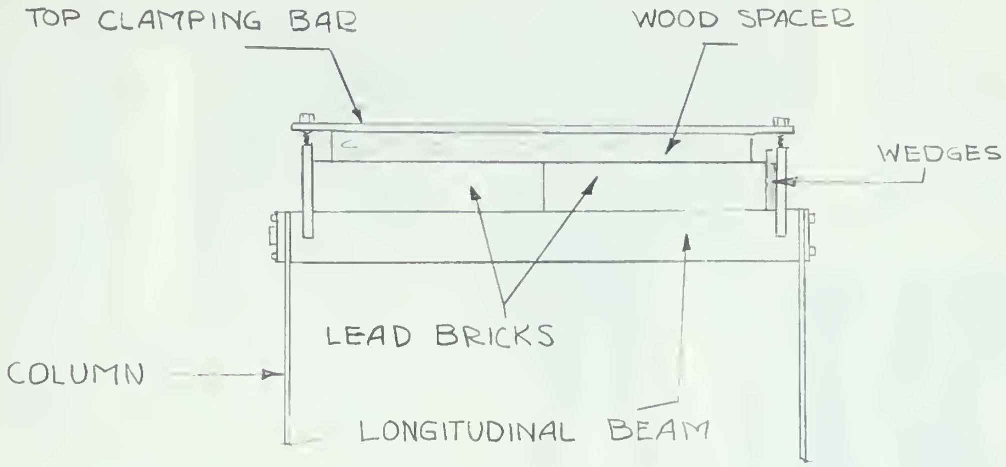

TOP CLAMPING BAR
WOOD SPACER
WEDGES
LEAD BRICKS
COLUMN
LONGITUDINAL BEAM

Fig 4b

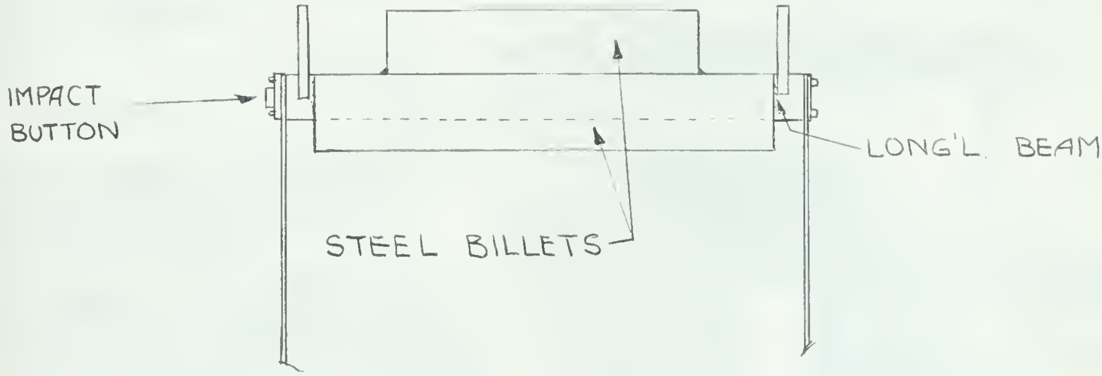

IMPACT
BUTTON
LONG'L. BEAM
STEEL BILLETS

Fig. 4c

Fig. 5a Model and Various Strikers

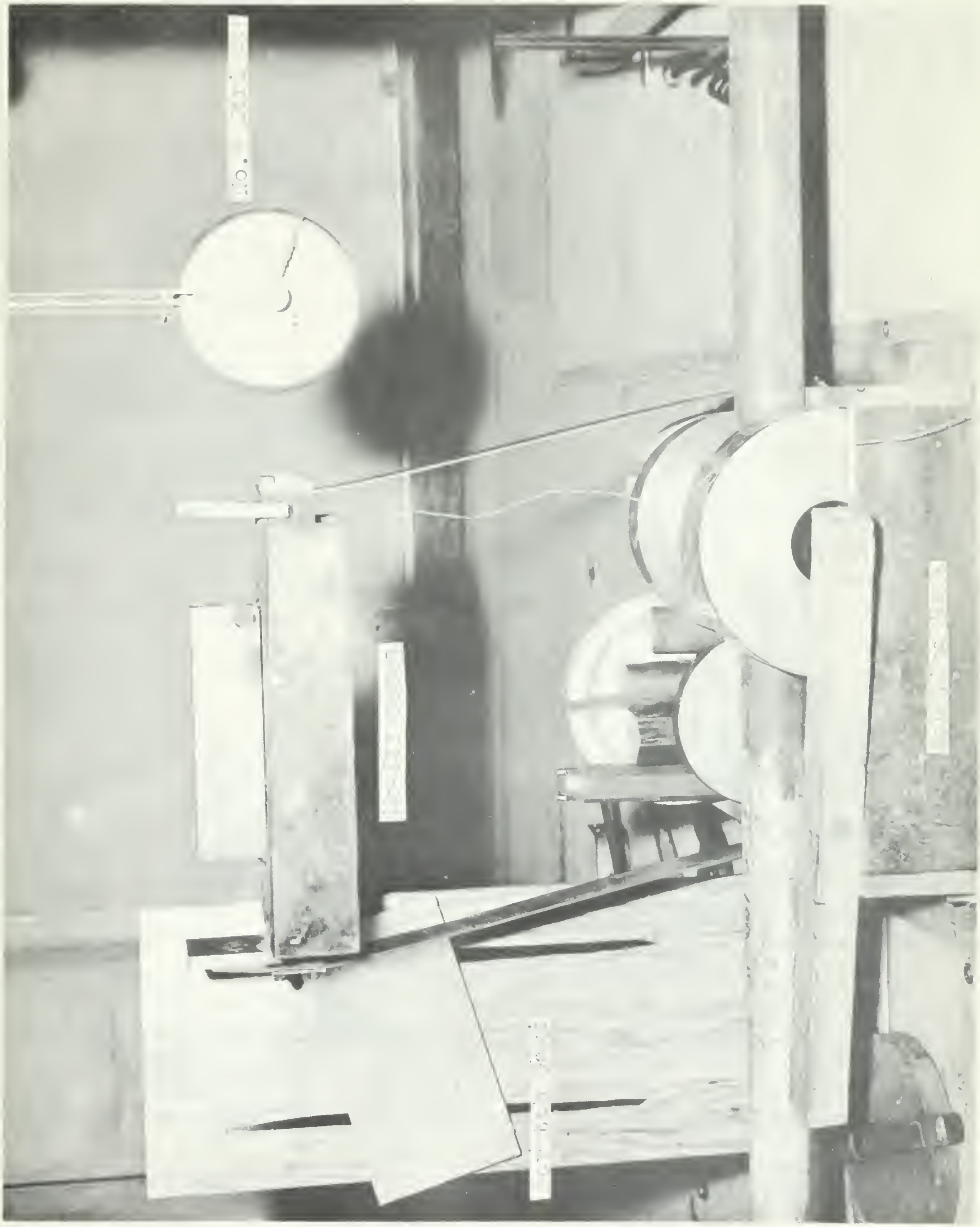


Fig. 5b Model and Fender after Impact

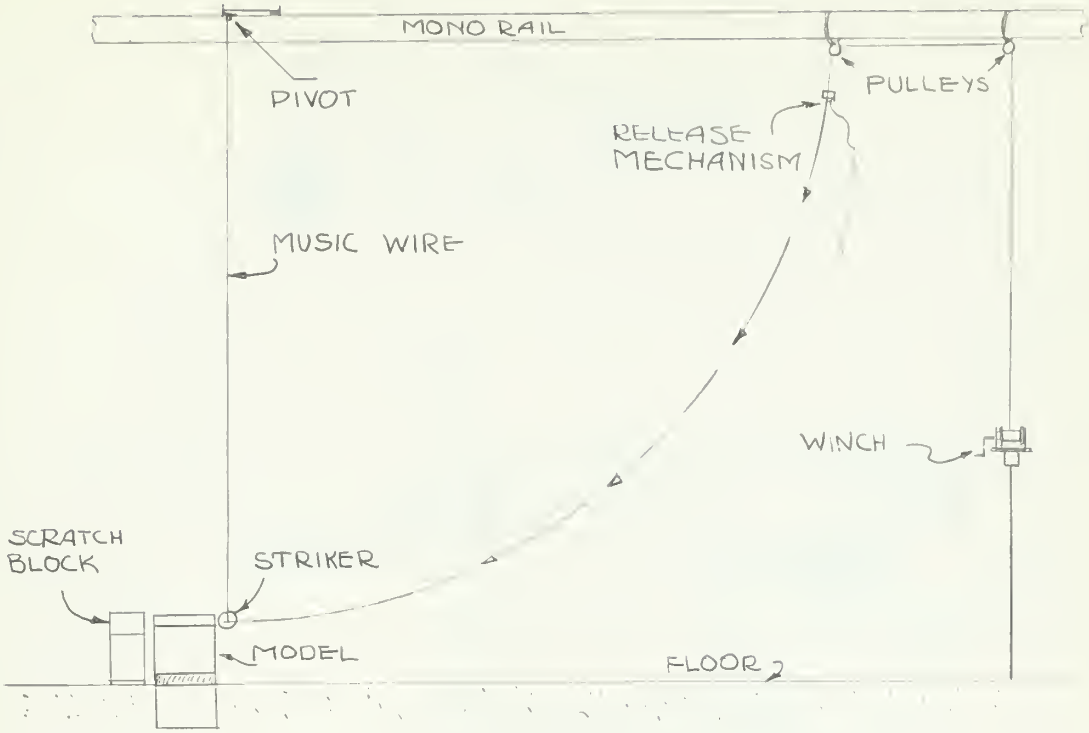


Fig. 6a

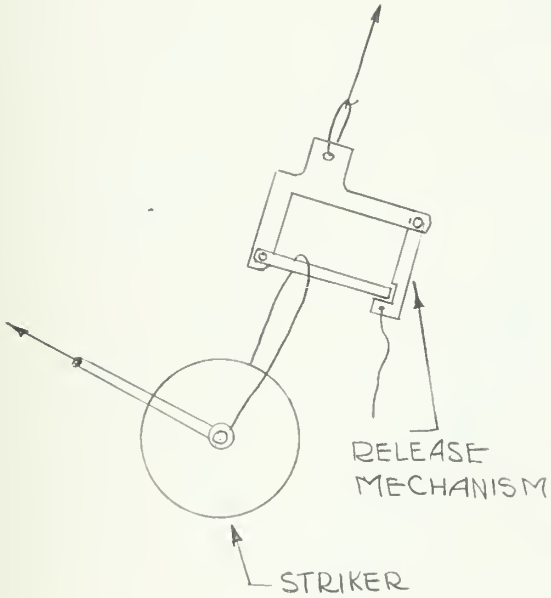


Fig. 6b

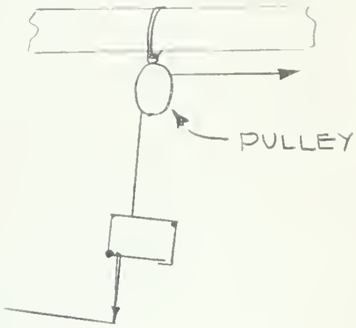


Fig. 6c

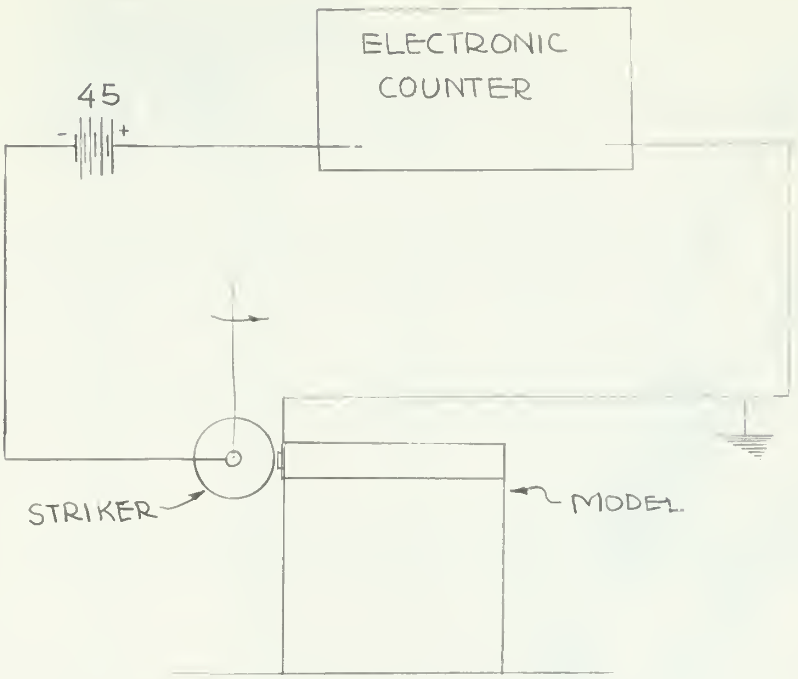


Fig. 7

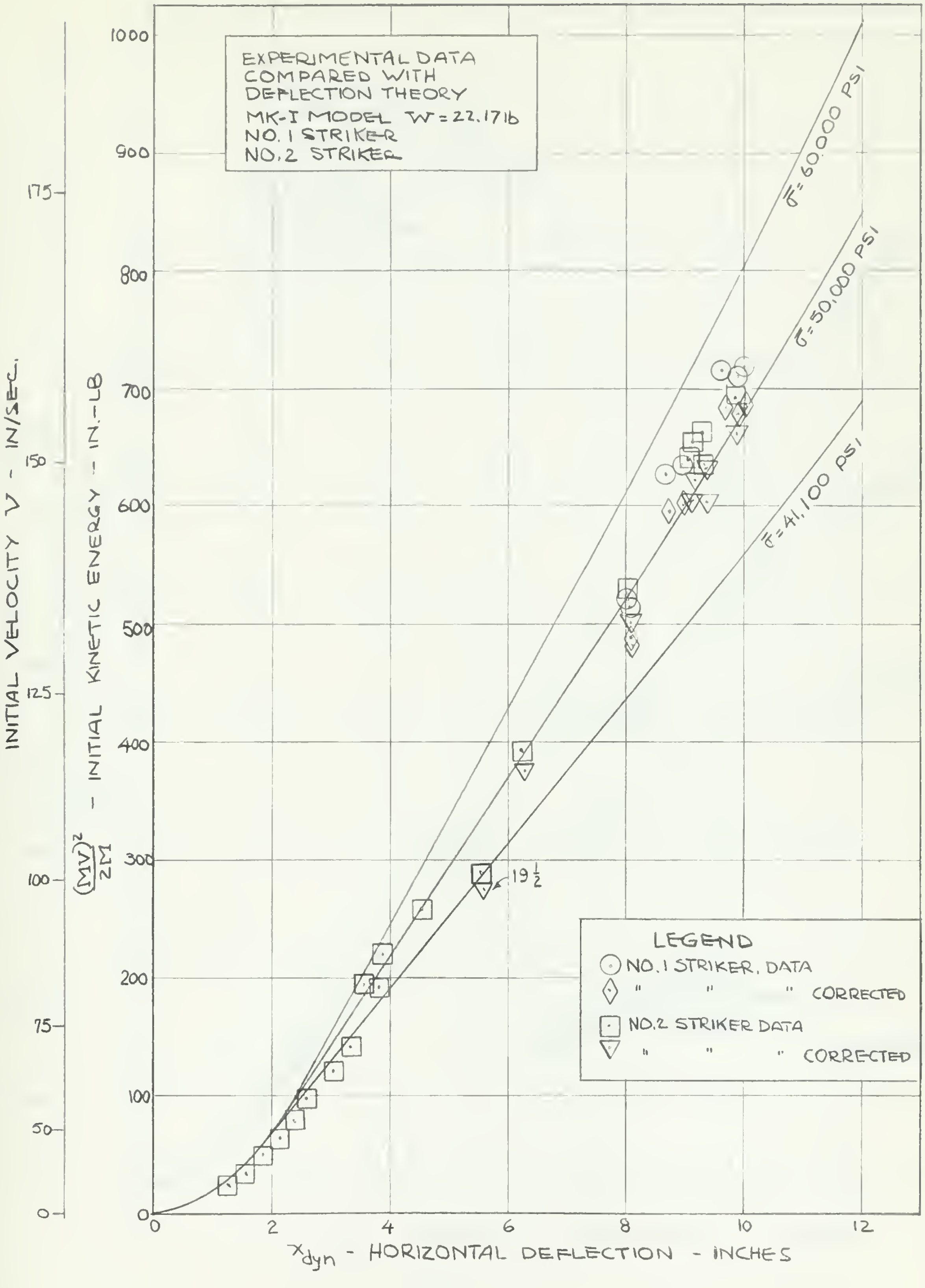

EXPERIMENTAL DATA
COMPARED WITH
DEFLECTION THEORY
MK-I MODEL W = 22.17 lb
NO. 1 STRIKER
NO. 2 STRIKER
LEGEND
NO. 1 STRIKER, DATA
" " " CORRECTED
NO. 2 STRIKER DATA
" " " CORRECTED
σ̄ = 60,000 PSI
σ̄ = 50,000 PSI
σ̄ = 41,100 PSI
19½
INITIAL VELOCITY V - IN/SEC.
(MV)²/2M - INITIAL KINETIC ENERGY - IN.-LB
x_dyn - HORIZONTAL DEFLECTION - INCHES

Fig. 8

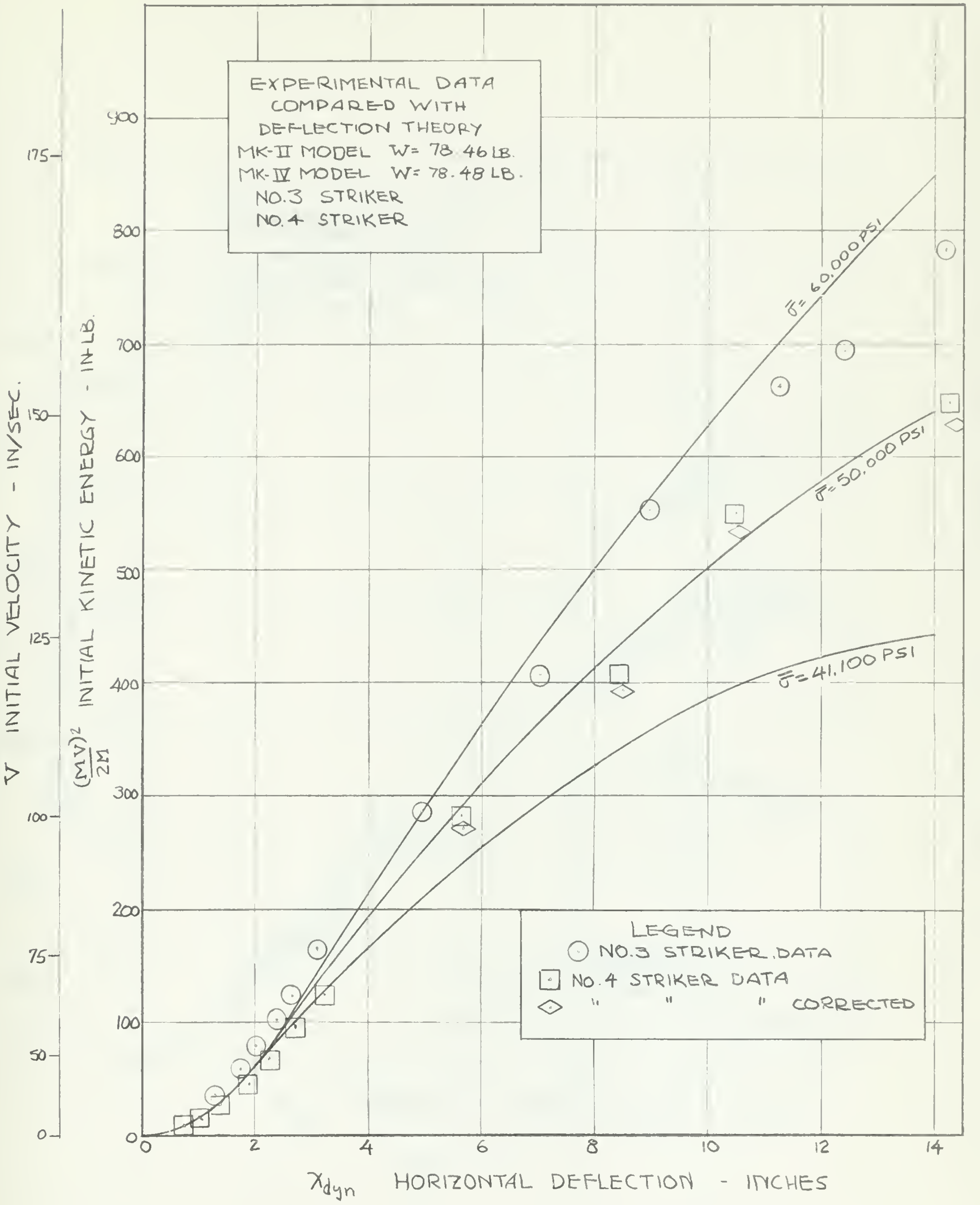

EXPERIMENTAL DATA
COMPARED WITH
DEFLECTION THEORY
MK-II MODEL W= 78.46 LB.
MK-IV MODEL W= 78.48 LB.
NO.3 STRIKER
NO.4 STRIKER
LEGEND
NO.3 STRIKER DATA
NO.4 STRIKER DATA
" " " CORRECTED
$\bar{\sigma}$= 60,000 PSI
$\bar{\sigma}$= 50,000 PSI
$\bar{\sigma}$= 41,100 PSI
V INITIAL VELOCITY - IN/SEC.
$\frac{(MV)^2}{2M}$ INITIAL KINETIC ENERGY - IN-LB.
X_{dyn} HORIZONTAL DEFLECTION - INCHES

Fig. 9

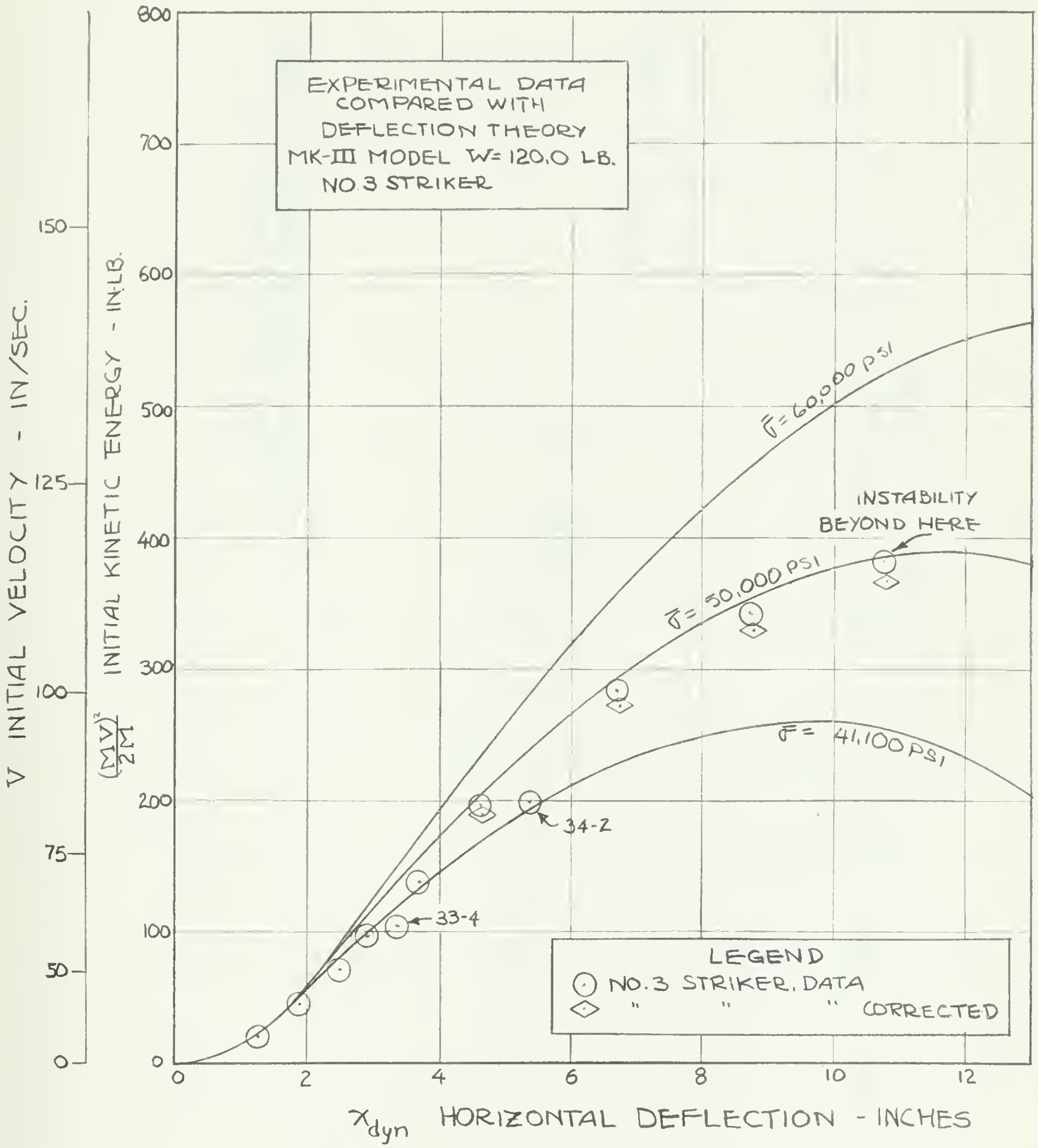

EXPERIMENTAL DATA
COMPARED WITH
DEFLECTION THEORY
MK-III MODEL W= 120.0 LB.
NO.3 STRIKER
$\bar{\sigma}$ = 60,000 PSI
INSTABILITY
BEYOND HERE
$\bar{\sigma}$ = 50,000 PSI
$\bar{\sigma}$ = 41,100 PSI
34-2
33-4
LEGEND
NO.3 STRIKER. DATA
" " " CORRECTED
V INITIAL VELOCITY - IN/SEC.
$\frac{(MV)^2}{2M}$ INITIAL KINETIC ENERGY - IN-LB.
x_{dyn} HORIZONTAL DEFLECTION - INCHES
0
50
75
100
125
150
0
100
200
300
400
500
600
700
800
0
2
4
6
8
10
12

Fig. 10

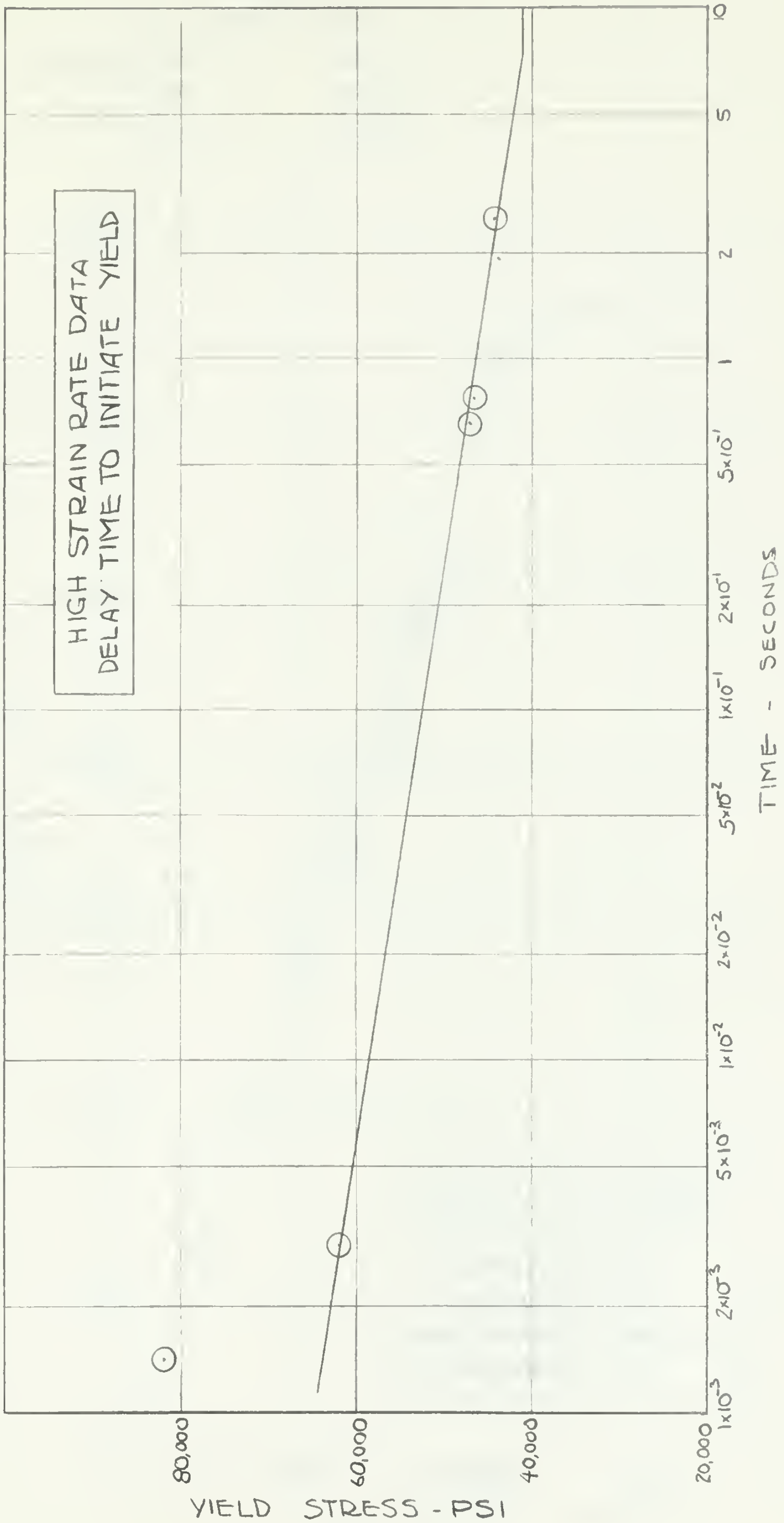

HIGH STRAIN RATE DATA
DELAY TIME TO INITIATE YIELD
TIME - SECONDS
1x10^-3
2x10^-3
5x10^-3
1x10^-2
2x10^-2
5x10^-2
1x10^-1
2x10^-1
5x10^-1
1
2
5
10
YIELD STRESS - PSI
20,000
40,000
60,000
80,000

Fig 11

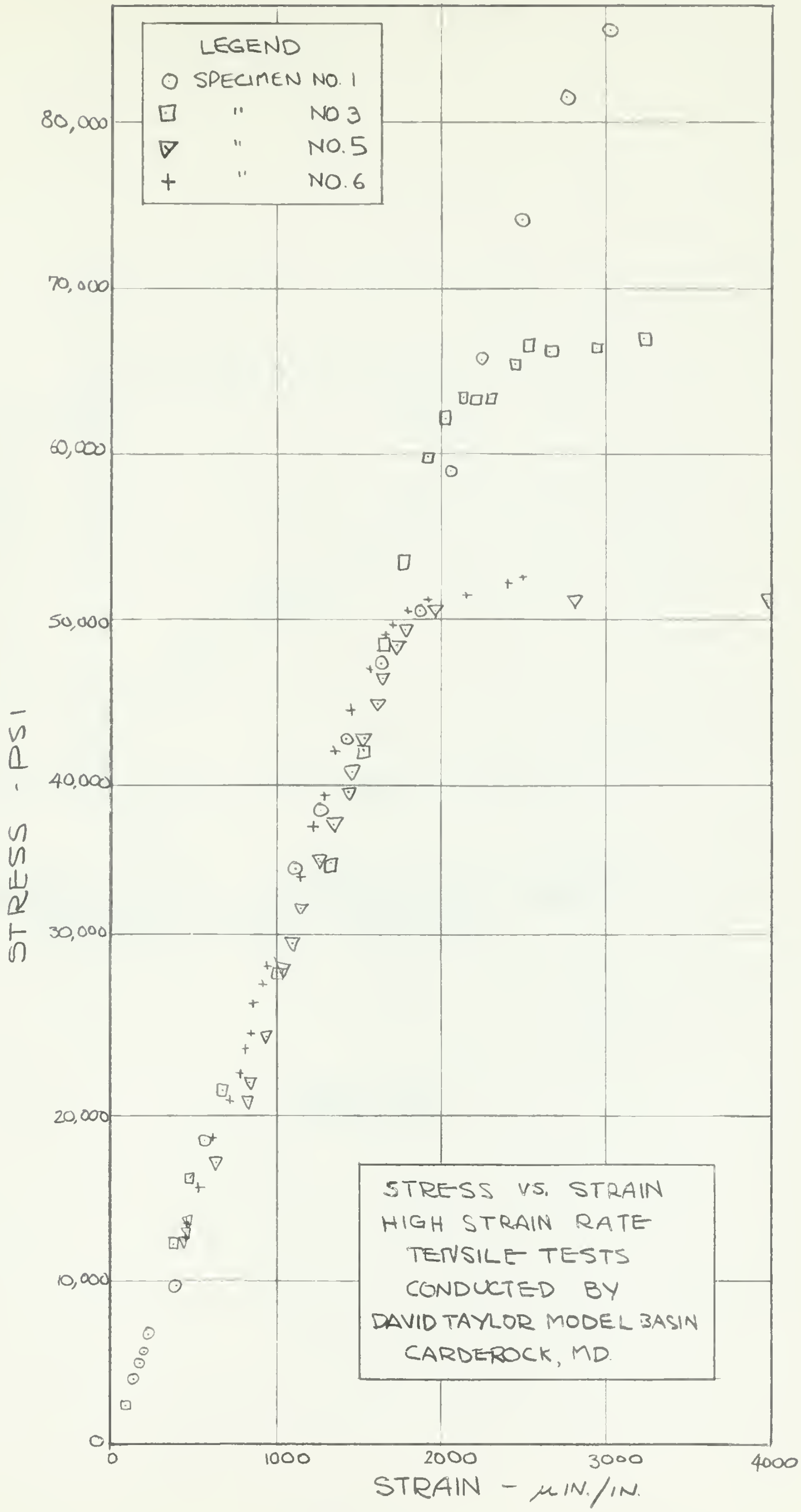
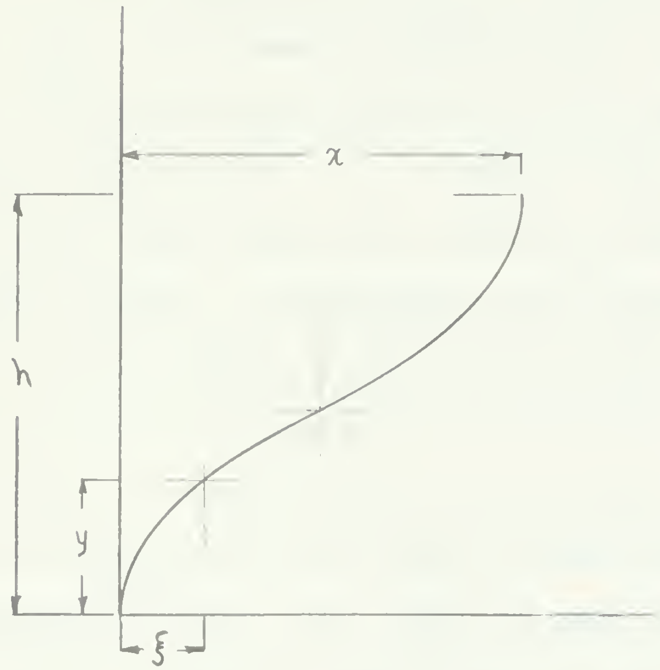

LEGEND
SPECIMEN NO. 1
" NO 3
" NO. 5
" NO. 6
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
STRESS - PSI
0
1000
2000
3000
4000
STRAIN - μIN./IN.
STRESS VS. STRAIN
HIGH STRAIN RATE
TENSILE TESTS
CONDUCTED BY
DAVID TAYLOR MODEL BASIN
CARDEROCK, MD.

Fig. 12



PRESUMED CURVATURE OF COLUMN
DURING INITIAL BENDING

Fig. 13

Appendix I Theoretical Considerations for the Effective Mass (M) of the Moving Structural Model

It must be made clear that the effective mass of the structural model for dynamical purposes and the weight of the longitudinal beam for structural purposes are not at all equivalent. As shown in Appendix III, the weights that influence the static determination of the large deflection structural theory are $W + w$. However, for a body in motion the two forces that affect the motion are proportional to the masses involved. Accordingly one should derive some relation for the effect of the masses of the structure on the motion of the structure.

It should be apparent that all portions of the model that can be considered to be part of the longitudinal beam (the fastenings, bolts, etc., all of which weigh W) certainly will move with the same acceleration due to the impulse applied. However, the columns being the radii on which the longitudinal beam rotates, accelerate in relation to their distance from the fixed end. It would be easy to assume that the columns do rotate as a straight rod on a pivot. However, the columns due to their elastic deflection (initially) and the axial load actually assume a curved shape. Such curvature means that the effect of the mass of the column in motion will not be linear with the distance from the base. It is dependent on the curvature. The procedure then is to assume a shape of curvature and

obtain a "reduced mass" which can be added to W/g which will give the effective mass M that is being moved by the impulse.

Refer to Fig. 13

Let: x = horizontal displacement at top

$\dot{x} = \frac{dx}{dt}$

y = vertical location of any particle in the column

$\xi = f(y)x$

$\dot{\xi} = f(y)\dot{x}$

w = weight of one column

m_{eq} = equivalent or reduced mass of the column

$$K.E. = \tfrac{1}{2}\int_0^h \dot{\xi}^2 \frac{w}{g}\frac{dy}{h} = \frac{w\dot{x}^2}{2gh}\int_0^h f^2(y)\,dy \qquad \text{Eq. 1}$$

also,

$$K.E. = \frac{1}{2} m_{eq}\dot{x}^2 = \frac{1}{2}\frac{w_{eq}}{g}\dot{x}^2 \qquad \text{Eq. 2}$$

Set Eq 1 = Eq. 2,

$$w_{eq} = \frac{w}{h}\int_0^h f^2(y)\,dy$$

First assume that the curvature of the columns is equivalent to two parabolas back to back.

$$f(y) = 2\left(\frac{y}{h}\right)^2 \qquad \text{for } 0 \le y \le \frac{h}{2}$$

$$f(y) = 1-2\left(\frac{h-y}{h}\right)^2 \qquad \text{for } \frac{h}{2} \le y \le h$$

Substituting:

$$w_{eq} = \frac{w}{h}\left\{\int_0^{\frac{h}{2}} \frac{4y^4}{h^4}\,dy + \int_{\frac{h}{2}}^{h}\left[1 - 4\left(\frac{h-y}{h}\right)^2 + 4\left(\frac{h-y}{h}\right)^4\right]dy\right\}$$

On integrating between limits it is found that,

$$w_{eq} = w(.383)$$

So, for two columns

$$w_{eq} = .766\ w$$

Total Kinetic Energy of the moving model is:

$$K.E._{total} = \frac{1}{2g}\left(W + .766\ w\right)V^2$$

Therefore, $M = (W + .766w)$

The second assumption is that the curvature of the moving column is of a sinusoidal nature.

Let: $f(y) = \sin^2\left(\frac{\pi y}{2h}\right)$

Following the previous procedure,

$$w_{eq} = \frac{w}{h}\int_0^h \sin^4\left(\frac{\pi y}{2h}\right)dy$$

On integrating between limits it is found that,

$$w_{eq} = w\left(\frac{3}{8}\right)$$

Total Kinetic Energy of the moving model (beam and two columns) is:

$$K.E._{total} = \frac{1}{2g}\left(W + .750\ w\right)V^2$$

Therefore,

$$M = W + \frac{3}{4}w$$

There are available two "reduced masses" which are reasonably close to each other. Actually both are gross approximations based on an assumed curvature of bending of the columns. One may be no better than the other. For calculations in this thesis the latter solution, wherein the column's reduced mass is $\frac{3}{8}\frac{w}{g}$, is arbitrarily chosen.

Appendix II Details of Steel used in the Columns

(a) The steel used was a 2" x 1/8" hot rolled strip to meet ASTM A303-58T standards. Measurement at some 25 locations showed that the average width of the steel was 1.97". The few variations from this width were $\pm$ 0.01".

The thickness of the strip averaged 0.120". The variation in thickness was $\pm$ 0.02" with the majority of the variations $< \pm$ 0.01".

The material supplied by Coulter Steel and Forge Co., was certified to be all of heat No. 162381. It was delivered in sawed, ten foot long strips.

(b) Partial Chemical Analysis of this steel is as follows:

Carbon 0.16%	Phosphorous 0.014%
Manganese 0.36%	Sulphur 0.019%

All of this is within the limits specified by the standards.

(c) Four samples of the steel strip were selected at random. These were then fashioned into standard tensile test specimens with a 2" gage length to meet ASTM Standard A307-54T.

The tensile tests were conducted on a Riehle 60,000 lb Universal Testing Machine. The first two specimens were tested using Riehle extensometers. The last two were instrumented with A-1 strain gages. The results are: an average modulus of elasticity of 29.9 x 10^6 psi; and an

average upper yield stress $\bar{\sigma}$ = 41,100 psi and an average lower yield stress $\bar{\sigma}$ = 39,700 psi. See Table 1 for test summary.

Table 1

Tensile Test Summary

	Specimen				
	1	2	3	4	Avg.
Modulus of Elasticity, psi	–	30.0×10^6	29.8×10^6	29.8×10^6	29.9×10^6
Upper Yield Stress, psi	40,900	40,500	42,500	40,500	41,100
Lower Yield Stress, psi	39,300	—	40,600	39,300	39,700
Ultimate Stress, psi	57,700	57,700	60,000	59,100	58,900

Note: Specimens #3 and #4 were tested with A-1 Strain Gages

Appendix III Tabulation of Experimental Data and Sample Calculation

STRIKER	STRIKER WEIGHT AND FASTENINGS, lb	m $\frac{lb. sec.^2}{in}$ (NOTE 1)
No. 1	6.57	0.01702
No. 2	7.11	0.01842
No. 3	22.90	0.0593
No. 4	17.22	0.0466

STRUCTURAL MODEL	W WEIGHT OF BEAM, FASTENINGS, lb	M $\frac{lb. sec.^2}{in.}$ (NOTE 2)
MK-I	20.92	0.0566
MK-II	78.46	0.2055
MK-III	120.0	0.311
MK-IV	78.48	0.2055

w = weight of one column, 18" long = 1.22 lb.

Note 1: m is the effective mass of the striker including the contribution of the fastenings, suspension and even the turnbuckles (in the No. 1 striker) as discussed in Part 3 of the Thesis.

Note 2: M is the effective structural mass of the structural model in motion.
It is different from W, in that it includes the effect of the mass of the columns. $M = \left[W + \frac{3}{8}(2w)\right]\left(\frac{1}{g}\right)$

See Appendix I for further explanation.

Sample Calculations of Experimental Data

For this purpose use data from Mk-II model Serial 15 test

Basic data, Experimental Measurements:

$h_1 = 212.0''$ $\quad$ $h_2 = 4.2''$

$x_{dyn} = 9.31''$ $\quad$ $x_{st} = 6.83''$

$M = 0.0566$ lb-sec^2/in. $\quad$ $m = 0.01842$ lb-sec^2/in.

$g = 386$ in/sec^2

$$v_1 = \sqrt{2gh_1} = 405 \text{ in/sec}$$

$$mv_1 = 7.45 \text{ lb-sec}$$

$$v_2 = \sqrt{2gh_2} = -56.8 \text{ in/sec}$$

$$mv_2 = -1.046 \text{ lb-sec}$$

$$MV = mv_1 - mv_2 = 7.45 - (-1.046) = 8.496 \text{ lb-sec}$$

$$\frac{(MV)^2}{2M} = \frac{(8.496 \text{ lb-sec})^2}{2(0.0566 \text{ lb-sec}^2/\text{in.})} = 636 \text{ in-lb.}$$

Tabulation of Experimental Data

MK-I Model Striker No.1

Model $M = 0.0566$ lb-sec^2/in. Striker No. 1 $m = 0.01702$ lb-sec^2/in.

Model Serial	h_1 in.	v_1 in/sec	mv_1 lb-sec	h_2 in.	v_2 in/sec	mv_2 lb-sec	MV lb-sec	$\frac{(MV)^2}{2M}$ lb-sec	x_{dyn} in.	x_{st} in.	y_{dyn} in.	y_{st} in.	T 10^{-6} sec	Corrections in-lb Down	Corrections in. Right
3	211.3	405.0	6.89	2.75	-46.1	-0.78	7.67	521	8.02	5.25	1.89	0.72	510	32	.05
4	211.6	405.0	6.89	10.7	-90.8	-1.55	8.44	628	8.69	6.13	2.48	1.17	400	32	.05
5	209.8	403.0	6.86	11.6	-94.5	-1.61	8.47	635	8.94	6.59	2.77	1.33	550	32	.05
6															
7	210.9	404.0	6.88	19.6	-123.0	-2.09	8.97	710	9.88	7.44	3.27	1.80	280	32	.05
8	210.8	404.0	6.88	2.66	-44.5	-0.76	7.64	515	8.06	5.34	2.19	0.97	460	32	.05
9	210.6	404.0	6.88	20.4	-125.4	-2.13	9.01	716	9.63	7.31	3.06	1.66	430	32	.06
10	210.9	404.0	6.88	20.6	-126.2	-2.15	9.02	718	10.00	7.69	3.31	1.84	260	32	.06
11	209.7	403.0	6.86	0.38	-17.0	-0.29	7.15	452	—	2.00	—	0.16	450	32	—

Tabulations of Experimental Data

MK-I Model Striker No.2

Model $M = 0.0566$ lb-sec^2/in. Striker No.2 $m = 0.01842$ lb-sec^2/in.

Model Serial	h_1 in.	V_1 in/sec	mV_1 lb-sec.	h_2 in.	V_2 in/sec	mV_2 lb-sec	MV lb-sec	$\frac{(MV)^2}{2M}$ lb-sec	x_{dyn} in.	x_{st} in.	y_{dyn} in.	y_{st} in.	T 10^{6} sec.	Corrections in-lb Down	Corrections in. Right
12	212.0	405.0	7.45	4.5	-59.0	-1.09	8.54	642	9.06	6.72	2.74	1.41	450	32	.06
13	212.0	405.0	7.45	5.25	-63.8	-1.18	8.63	655	9.13	6.63	2.69	1.36	490	32	.06
14	212.0	405.0	7.45	7.6	-76.8	-1.41	8.86	693	9.81	7.39	3.09	1.66	500	32	.06
15	211.9	405.0	7.45	4.2	-56.8	-1.05	8.50	636	9.31	6.83	2.75	1.44	390	32	.06
16	214.8	407.5	7.50	5.2	-63.3	-1.17	8.67	664	9.31	6.78	2.75	1.36	400	32	.06
17	160.9	352.5	6.49	5.7	-66.4	-1.22	7.71	526	8.02	5.34	1.97	0.83	640	25	.05
18	115.8	299.0	5.51	5.2	-63.3	-1.17	6.68	393	6.25	3.53	1.17	0.36	340	18.5	.04
19	77.4	244.5	4.50	3.7	-53.4	-0.98	5.48	265	–	1.95	–	0.11	390	13.5	–
19½	77.4	244.5	4.50	5.4	-64.8	-1.19	5.69	287	5.56	2.56	0.95	0.20	370	13.5	.03
20-1	6.69	71.9	1.33	0.44	-18.7	-0.34	1.67	24.3	1.28	0	0.03	0	–	5	–
20-2	8.81	82.5	1.52	0.69	-23.1	-0.42	1.94	33.2	1.56	0	0.06	0	–	5	–
20-3	12.31	97.6	1.80	1.3	-31.8	-0.58	2.38	50.1	1.84	0	0.09	0	650	5	–
20-4	16.19	111.8	2.06	1.6	-34.8	-0.64	2.70	64.4	2.13	0	0.13	0	340	5	–
20-5	20.19	124.9	2.30	1.7	-36.1	-0.67	2.97	77.6	2.38	0	0.16	0	350	5	–
20-6	25.19	139.3	2.57	2.1	-40.5	-0.75	3.31	96.9	2.59	0.03	0.19	0	350	7	–
20-7	28.94	149.5	2.76	3.3	-50.6	-0.93	3.69	119.7	3.03	0.22	0.28	0	330	7	–
20-8	35.81	166.3	3.06	3.4	-51.5	-0.95	4.01	141.5	3.34	0.36	0.34	0	420	9	–
20-9	52.31	201.0	3.71	3.6	-52.9	-0.98	4.68	193.4	3.81	0.81	0.56	0.05	460	10	–
21	65.6	224.0	4.13	1.2	-30.3	-0.56	4.69	194.0	3.56	0.75	0.41	0.03	–	12	–
22	67.5	229.0	4.21	2.2	-41.7	-0.77	4.98	219.0	3.88	6.96	–	–	340	12	0.02
23	76.9	244.0	4.49	3.3	-50.6	-0.93	5.42	259.0	4.53	1.81	0.63	0.06	–	13.5	0.02

N.B. Test 19½ is a retest of Model 19. Tests 20-2 thru 20-9 are all retests of Model 20-1

Tabulation of Experimental Data

MK-II Model Strikers No.2 and No.3

MODEL $M = 0.2055$ lb-sec²/in. STRIKER No.3 $m = 0.0466$ lb-sec²/in.

MODEL SERIAL	h_1 in.	V_1 in/sec	mV_1 lb-sec	h_2 in.	V_2 in/sec.	mV_2 lb-sec	MV lb-sec	$\frac{(MV)^2}{2M}$ lb-sec	x_{dyn} in.	x_{st} in.	y_{dyn} in.	y_{st} in.	T 10^{-6} sec
25-1	5.31	64.1	3.80	0	0	0	3.80	35.1	1.38	0	0.06	0	-
25-2	8.69	82.0	4.86	0	0	0	4.86	57.5	1.75	0	0.09	0	-
25-3	11.88	95.9	5.68	0	0	0	5.68	78.6	2.03	0	0.14	0	-
25-4	15.31	108.8	6.46	0	0	0	6.46	101.2	2.38	0.03	0.19	0	-
25-5	18.63	120.0	7.12	0	0	0	7.12	123.3	2.66	0.13	0.25	0.02	-
25-6	25.19	139.5	8.27	0	0	0	8.27	165.9	3.09	0.56	0.33	0.02	-
26	49.9	196.3	11.67	0.25	+ 13.9	+0.82	10.85	286	4.94	3.00	0.73	0.27	-
27	74.9	240.5	14.23	0.63	+ 22.0	+ 1.30	12.93	407	7.03	5.63	1.59	1.00	477
28	96.6	273.0	16.18	0.44	+ 18.4	+ 1.09	15.09	553	8.94	8.03	2.56	2.06	530
29	124.5	310.5	18.38	0.81	+ 25.1	+ 1.48	16.90	694	12.41	11.94	5.47	5.00	640
30	154.2	346.5	20.53	2.44	+ 43.4	+ 2.57	17.96	783	14.19	13.94	7.72	7.34	-
31	175.2	368.0	21.80	2.44	+ 43.4	+ 2.57	19.23	900	-	16.50	-	-	-
32	121.9	307.0	18.20	1.06	+ 27.8	+ 1.65	16.55	663	11.25	10.63	4.31	3.75	2870

STRIKER No.2 $m = 0.01842$ lb-sec²/in.

24	202.1	395.5	7.30	6.19	-69.1	-1.27	8.57	178.3	-	1.44	-	0.06	—

N.B TESTS 25-2 THRU 25-6 ARE ALL RETESTS OF MODEL 25-1

Tabulation of Experimental Data

MK-III Model Striker No. 3

Model $M = 0.311$ lb-sec^2/in. Striker No. 3 $m = 0.0593$ lb-sec^2/in.

Model Serial	h_1 in.	V_1 in/sec	mV_1 lb-sec	h_2 in.	V_2 in/sec	mV_2 lb-sec	MV lb-sec.	$\frac{(MV)^2}{2M}$ in-lb	X_{dyn} in.	X_{st} in.	Y_{dyn} in.	Y_{st} in.	T 10^{-6} sec	Corrections in-lb Down	Corrections in. Right
33-1	2.50	44.0	2.61	0.38	-17.0	-1.01	3.62	21.1	1.28	0	0.03	0	-	0	0
33-2	5.50	65.2	3.86	0.75	-24.1	-1.43	5.29	44.8	1.88	0	0.13	0	2320	0	0
33-3	9.63	86.3	5.11	0.88	-26.0	-1.54	6.65	71.2	2.50	0	0.22	0	610	0	0
33-4	13.38	101.6	6.03	1.50	-34.1	-2.02	8.05	104.1	3.38	0.38	0.34	0	710	0	0
33-5	18.75	120.3	7.14	1.63	-35.4	-2.10	9.24	136.9	3.72	1.41	0.56	0.09	560	0	0
34-1	10.63	90.6	5.38	2.19	-40.0	-2.37	7.75	96.5	2.94	0.25	0.31	0	70	0	0
34-2	27.31	145.3	8.61	2.31	-42.3	-2.51	11.12	199.7	5.41	3.69	1.00	0.50	90	8	0.03
35	37.81	171.0	10.13	3.69	-53.4	-3.16	13.29	283.0	6.72	5.88	1.50	1.09	500	11	0.04
36	47.8	192.2	11.39	3.81	-54.3	-3.22	14.61	342.0	8.75	8.38	2.50	2.25	600	13	0.05
37	52.7	202.0	11.97	5.72	-58.3	-3.45	15.42	382.0	10.75	10.75	3.59	3.59	640	15	0.06
38	26.56	143.2	8.49	2.44	-43.4	-2.57	11.06	196.2	4.66	2.88	0.66	0.22	520	7	0.02

N.B. Tests 33-2 thru 33-5 are retests of model 33-1. Test 34-2 is a retest of model 34-1

Tabulation of Experimental Data

MK-IV Model Striker No.4

MODEL $M = 0.2055$ lb-sec^2/in. STRIKER No.4 $m = 0.0466$ lb-sec^2/in.

MODEL SERIAL	h_1 in.	V_1 in/sec	mV_1 lb-sec	h_2 in.	V_2 in/sec	mV_2 lb-sec	MV lb-sec	$\frac{(MV)^2}{2M}$ in-lb	x_{dyn} in.	x_{st} in.	y_{dyn} in.	y_{st} in.	T 10^6 sec
39-1	1.00	27.8	1.24	0.25	-13.9	-0.62	1.86	8.4	0.75	0	0.03	0	1150
39-2	2.00	40.6	1.81	0.44	-18.4	-0.82	2.63	16.8	1.03	0	0.06	0	1020
39-3	3.56	52.5	2.34	0.56	-20.9	-0.93	3.27	26.0	1.38	0	0.09	0	720
39-4	6.75	72.7	3.24	0.75	-24.1	-1.07	4.31	45.2	1.88	0	0.16	0	650
39-5	10.38	89.6	3.99	1.00	-27.8	-1.24	5.23	66.6	2.25	0	0.22	0	660
39-6	15.50	109.5	4.88	1.25	-31.1	-1.39	6.27	95.7	2.72	0.06	0.31	0	630
39-7	19.75	123.5	5.51	1.75	-36.8	-1.64	7.15	124.2	3.22	0.56	0.41	0.03	830
40	47.75	192.0	8.56	3.13	-49.1	-2.19	10.75	281	5.66	3.63	1.03	0.41	600
41	97.09	274.0	12.21	5.22	-63.5	-2.83	15.64	551	10.50	9.72	3.72	3.16	700
42	114.25	297.5	13.27	6.09	-68.6	-3.06	16.33	648	14.25	13.94	7.81	7.41	780
43	72.59	237.0	10.58	3.59	-52.7	-2.35	12.93	407	8.44	7.41	2.31	1.72	820

N.B: TESTS 39-2 THRU 39-7 ARE ALL RETESTS OF MODEL 39-1

Appendix IV Energy - Strain Deflection Calculations

As discussed in Section 2, titled "Background for Investigation," it is more accurate to employ the large deflection theory to predict the behavior of the structural models during their deformations. Small deflection theory, in which simplifications such as $x \approx \sin x$ are employed, would involve appreciable errors for deflections greater than four inches. The basic equations are developed in Section 2.

Equations F and G are not easy to solve for x_{dyn} knowing the initial kinetic energy of the structural model. So the method of solution here was to solve for kinetic energy from a given x. This allowed development of large deflection theory curves in Figs. 8, 9, and 10 against which the experimental data may be compared.

A sample calculation follows:

Basic Data

$\overline{\sigma} = 41{,}100$ psi

$x = 6.0"$

$L = 18.0"$

$E = 29.9 \times 10^{6}$ psi

$I = \frac{bh^3}{12} = 284 \times 10^{-6}$ in.^{4}

$w = 20.92$ lb.

$w = 1.22$ lb.

$b = 1.97"$ (width of column)

$h = 0.120"$ (Thickness of column)

$$M_u = \frac{\bar{\sigma}\, bh^2}{4} = \frac{(41,100)\ \text{lb/in}^2\ (0.97)\ \text{in}\ (0.170)^2\ \text{in}^2}{4}$$

$$= 292\ \text{in-lb}$$

$$P_a = \frac{(W + w)}{2} = 11.07\ \text{lb}$$

$$P_e = \frac{\pi^2 EI}{L^2} = 258.5\ \text{lb.}$$

From Eq. F for $R_1(x)$ and from Eq. G for $R_2(x)$ we solve simultaneously to find x_u (the horizontal deflection at which the Fully Plastic Bending Moment is first attained).

$$\frac{24EI\ x}{L^3}\left(1 - \frac{P_a}{P_e}\right) = \frac{4M_u - (W+w)\,x}{\sqrt{L^2 - x^2}}$$

$$\frac{(24)(29.9)10^6(284)10^{-6}\ x}{(18)^3}\left(1 - \frac{11.07}{258.5}\right) = \frac{4(292) - (22.14)\,x}{\sqrt{18^2 - x^2}}$$

Thus, $\quad x_u = 1.876''$

Now solve for the Kinetic Energy at any x (but, in this example, for $x_{dyn} = 6.0''$)

$$K.E. = \frac{12EI}{L^3}\left(1 - \frac{P_a}{P_e}\right)x_u^2 + 4M_u \sin^{-1}\frac{x_{dyn}}{L} - \left(L - \sqrt{L^2 - x_{dyn}^2}\right)(W+w)$$

$$- \left[4M_u \sin^{-1}\frac{x_u}{L} - \left(L - \sqrt{L^2 - x_u^2}\right)(W+w)\right]$$

$$K.E. = 314\ \text{in-lb}$$

Deflection Theory Tabulation of Computed Data

MK I Structural Model

x (in.)	Kinetic Energy (in.-lb.) $\bar{\sigma}$ = 41,100 psi	$\bar{\sigma}$ = 50,000 psi	$\bar{\sigma}$ = 60,000 psi
0.5	4.2	4.2	4.2
1.0	16.7	16.7	16.7
2.0	66.5	66.8	66.8
4.0	191.0	218.1	241.0
6.0	313.5	369.6	426.0
8.0	435.4	523.0	612.4
10.0	561.0	678.6	806.0
12.0	691.0	846.6	1012.0

MKII and MKIV Structural Model

0.5	3.7	3.7	3.7
1.0	14.75	14.75	14.75
1.5	32.2	32.2	32.2
2.0	58.9	59.0	59.0
4.0	165.7	192.5	213.4
6.0	253.7	311.5	363.9
8.0	326.1	413.4	500.8
10.0	385.7	503.0	628.4
12.0	424.7	579.0	743.4
14.0	444.2	640.5	850.0

MKIII Structural Model

0.5	3.3	3.3	3.3
1.0	13.4	13.4	13.4
1.5	30.0	30.0	30.0
2.0	53.3	53.4	53.4
4.0	145.5	170.9	191.0
6.0	211.5	265.9	319.5
8.0	248.1	334.0	421.6
10.0	259.8	375.7	500.3
12.0	234.6	387.7	551.3
14.0	165.8	360.7	571.3

Appendix V High Strain Rate Test Data

As a matter of scientific curiosity, it was decided to investigate the behavior of some samples of the steel used in the columns, under very rapid loadings. Accordingly the David Taylor Model Basin was asked to conduct such tests. Through the good offices of Mr. Glenn D. Elmer, the designer of the Model Basin's rapid load machine, the tests were accomplished.

Six samples were tested. Of these, the results of four, perhaps five, can be relied upon. The data obtained showed a true delay time before yielding began when the samples were rapidly loaded. The composite results of these tests are presented in Fig. 11, wherein the delay time is compared to the associated increase in the dynamic yield strength. An exponential relation used in discussing the present model test results is indicated by the straight line in Fig. 11. Fig. 12 shows the conventional stress-strain curves for four specimens as plotted from Model Basin test data.

Table 2 summarizes some of the more pertinent test results.

The uncertainty of the data, as stated by Model Basin personnel, was of the order of 3-4%.

Table 2

Test Summary of Results of High Strain Rate Tensile Tests Conducted at the Structural Mechanics Laboratory of the David Taylor Model Basin, Carderock, Maryland

	Specimen*				
	1	2	3	5	6
Dynamic Yield Stress, psi	82,000	46,500	62,000	44,000	47,000
Time Delay, sec.	0.00142	0.770	0.003	2.50	0.660
Rupture Stress, psi	104,000	–	65,000	–	–

*Test #4 was unsuccessful